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The transition in the growth rate

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Abstract:
Kernel regressions reveal a hump-shaped transition path in the growth rate explained by the level of income in cross-country/time-series data. At the low-income end countries diverge and at the high-income end they converge. A growth peak appears in the middle. This empirical result survives a number of robustness tests, but two major exceptions appear: (i) the divergence starts a bit above the minimum income level; (ii) very resource rich countries converge throughout to low growth. The transition path is simulated by a two-sector model, where the traditional sector is gradually replaced by the modern one.

Keywords: Kernel regressions, transitional growth, Grand Transition, two-sector model JEL code: O41, O47, C49

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1. Introduction

Few relations in growth empirics have been studied as much as the one from the income level, y, to the growth rate, g. The focus of the newer literature has been on the transition to the modern steady state, with the rate of convergence, β, as the parameter of interest. We expand this focus by considering *both ends* of the transition. *Low-income* countries *diverge* as they leave the traditional steady state at different times. *High-income* countries *converge* as they approach the same steady state. The path of the growth rate between the two ends has one peak in the middle. The transition curve is rather robust even when the level of income explains little of large variation in the growth rate. We have found two large exceptions: (i) The divergence starts at a level of about $1'100 – at lower income levels, the picture is unclear. (ii) The transition curve is falling throughout in very resource rich countries – they converge to low growth. These stylized facts ought to be uncontroversial, but in the last 2-3 decades, most of the large empirical literature on convergence has estimated log-linear relations, derived from the one-sector Solow model, which are not compatible with a hump-shaped transition curve in the growth rate.

A large literature shows that most (all?) variables have transitions with different levels in the traditional and the modern steady state. The transition in variable x is visible as the path of the *kernel regression*: $x = \kappa(\bar{y}_{-},\ bw)$, where bw is a band around the average for initial income $\bar{y}_{-}$. Transitions are a movement between levels that are stable at both ends. Thus, changes in the transition variable are small in the steady states and larger in-between. The growth rate is an aggregate of the changes in all transition variables. This suggests a hump-shaped transition in the growth rate precisely as we find.

The kernel, $g = \kappa(\bar{y}_{-}, bw)$, is interpreted as a causal relation, $y_{-} \Rightarrow g$, for two reasons: (i) y_{-} is the initial income for each period, so $g \Rightarrow y_{-}$ would mean that g explaines y the preceding year. (ii) The kernel regression replaces y_{-} with the mean $\bar{y}_{-}$ for the *band* around y_{-}. The band we use normally contains observations from about 25 different countries. It is hard to believe that they react to the growth rate in one of the countries.

Section 2 provides a few notes to the literature, while section 3 describes the data and the kernel specifications. Section 4 reports our main empirical results using kernel regressions. Section 5 is a survey of a large set of robustness experiments reported in the Appendix. Section 6 simulates the observed transition path by a two-sector model, and section 7 concludes. To make everything easy to replicate the calculations are done with Stata, and the income data are the *cgdppc*-series for 1950-2016 from the 2018 version of the Maddison Project Database.

2. Notes on the literature

The study of economic growth is a very large subject as witnessed by the four stately volumes of Aghion and Durlauf (2004, 2014). Section 2.1 looks back at the transition literature, while section 2.2 asks if the growth regression literature can explain the transition. Finally, section 2.3 discusses how the various theoretical ideas should look like in a kernel regression framework.

2.1 The transition literature

A transition is a path between two equilibria, i.e., steady states. We believe that the data points to the existence of two basic steady states: A traditional and a modern one. The transition between the two is the *Grand Transition*.[3] During the transition, the two steady states coexist as two sectors, where the modern sector is gradually replaced by the traditional sector. In the 1960s and 1970s, two-sector models were a key theme in development economics, with the transition rather than the steady states as the main topic of interest; see Lewis (1954), Rostow (1960) and Ranis and Fei (1961), but this perspective has almost disappeared from the literature.

A few recent attempts have tried to include both ends of the transition. Maddison (2001) showed that the traditional steady state as first described by Malthus (1798) prevailed throughout most of history. Average incomes were low, and incremental technology advances supported a slow but steady increase in population size, though not a persistent increase in per capita income. Unified growth theory (Galor 2005) modeled this insight by emphasizing latent human capital accumulation as an explanation for the shift from the traditional to the modern steady state. Finally, Lucas (2009) modernized the two sector-model; see section 6.

The revival of neoclassical growth theory since the 1980s has shifted the focus on the determinants of endogenous steady state growth (Romer 1990), where a key theme is that larger populations meant more ideas. Hence, population growth translated into a faster rate of technological change. The transition to the modern steady state with a persistent growth rate of per capita income in the range of 1½-2% first took off in a few Western countries and their offshoots about 2½ centuries ago.

Over the last 50 years, selected countries from the European periphery and in East Asia, notably Hong Kong, Singapore, South Korea, and Taiwan, have caught up with the high-income countries due to persistent high growth rates, but these selected success stories hardly qualify as

3. Both China and most of Sub Saharan Africa seem to have had zero economic growth between years 1000 and 1700, but the traditional technology was rather different. Even then, the difference in income was small, according to the brave assessment in Maddison (2001, p 264).

a general pattern of catching up and long-run development. In contrast, it has been claimed that *low level equilibrium traps* may keep countries in a traditional steady state (Aziarides and Stachurski 2005) or that two rather isolated peaks for rich and for poor countries dominate the world income distribution (Quah 1996).

2.2 The one sector Solow model and estimation techniques

Graphically the one sector Solow-model (Solow 1956) looks as the gray line in Figure 1 below. The model is often operationalized as a log-linear panel regression, where income y is in logs:[4]

(1) $g_{it} = \alpha + \beta\, y_{it-1}$, where β is the coefficient of convergence, which should be negative

It is a generally accepted result that (1) gives no robust empirical evidence for absolute β-convergence except in the high-income countries: on average, poor countries do not grow faster than rich ones. This has led to a large search for an augmented model and estimators that generate a negative estimate of β.

(2) $g_{it} = \alpha_{it} + \beta\, y_{it-1} + \{\gamma_1\, z_{1it} + .. + \gamma_j\, z_{jit}\}$, where the constant is be broken into fixed effects, and the control set {…} contains a selection from a large number of possible variables.

Here (2) is termed a conditional convergence equation. Not surprisingly, it is possible to find a wide range of estimates of β when experimenting with fixed effects, various z-variables and estimators. Equation (1) is certainly linear. But if (2) uses variables that take care of the non-linearity it does not ask whether poor countries are catching up with rich countries, but rather whether there is a common speed of convergence to country-specific steady states.[5]

By controlling for country- and year-fixed effects in addition to steady state determinants like factor accumulation and population growth, Islam (1995) has moved the discussion still further away from the initial research question on cross-country catching up and convergence. The concept of convergence has been further eroded by allowing for country-specific rates of technological change (Lee *et al*. 1998). With each "commonality" restriction eliminated from the data, the number of potential steady states will increase and may finally exceed the number of cross-section observations. Conditional convergence has become a concept with little relevance in discussions of a potential general pattern of long-run growth and development.

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4. The derivation of (1) from the Solow-model is found, e.g., in Barro and Sala-i-Martin (1992).
5. One of the first cross-country studies showing β-convergence (Baumol 1986) was criticized for limiting the sample to high-end countries (De Long 1988). For a broad sample of countries, evidence for the "iron law" (Barro 2015) of a theoretically predicted convergence rate of about 2% has only been found by assuming that countries are converging to different steady states.

In addition there has been a large effort to improve estimators: Many authors have used dynamic panel estimators like Difference-GMM (Caselli *et al.* 1996) and System-GMM (Bond *et al.* 2001). However, the Monte Carlo simulations by Hauk and Wacziarg (2009) have revealed the missing robustness of these estimators in the presence of the multiple data problems that abound in typical cross-country panel data sets. In addition, Eberhardt and Teal (2011) point out that the simulations ignore three important features of modern panel time-series econometrics, namely technology heterogeneity, variable non-stationarity and cross-section correlation. Thus, up to now, the perfect estimator has not been found.

Consequently, the present state of empirical research on transitional growth paths is rather unsatisfactory. Despite many methodical innovations over the last 20 years, there are not even robust estimates of the rate of conditional convergence, which is a concept that is at best weakly related to patterns of long-run development. The concept of conditional convergence comes close to a tautology by predicting that countries would transition to the same steady state if they were the same, except for their initial income. We claim the relevant question is whether countries, despite all differences, have enough commonality to give a common transitional growth-income path and hence a general pattern of long-run growth and development.

Moreover, the long-run transition of an economy from the traditional to the modern steady state is probably not the same process as the adjustment of an economy that is close to, but not in, its modern steady state. The latter problem motivated the original contribution by Solow (1956). Therefore, it becomes questionable why one should impose the log-linear Solow restriction on the specification of a growth equation to be estimated with cross-country panel data. We claim that the main reason why the empirical convergence literature has not found an unconditional log linear growth-income path is that the one-sector workhorse model of growth empirics is the wrong tool for studying the Grand Transition.

The data from Maddison Project Database (2018) reveal that per capita income growth took off in the early industrial economies in the 19th century, when average incomes were already substantially higher than at the beginning of the Industrial Revolution. If high growth is more common midways in the transition, the modeling of a hump-shaped transitional growth-income path should become the norm.[6]

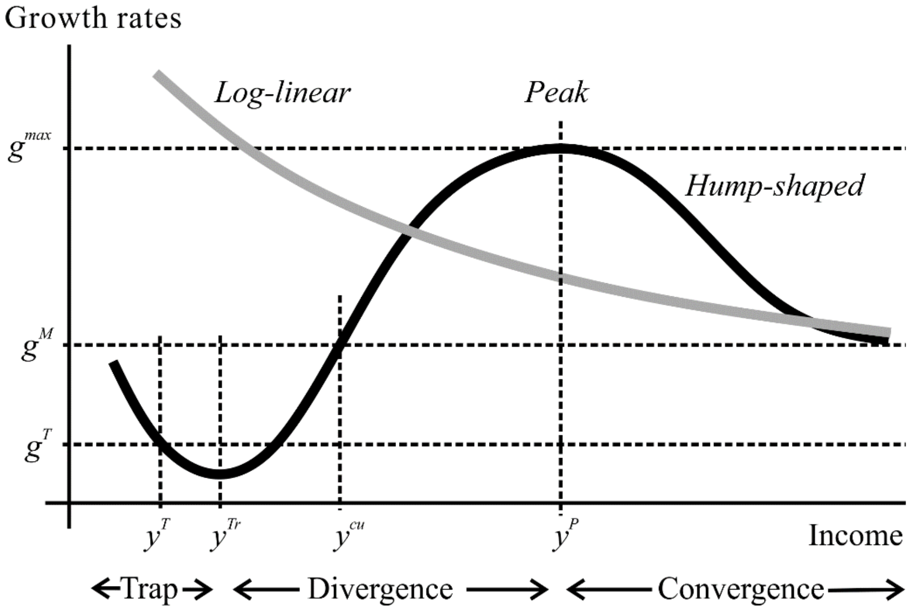
2.3 A graphical survey: what should we look for?

Our theoretical discussion of transitional growth paths is summarized on Figure 1, with income,

6. Inspired by Rostow (1960) Laursen and Paldam (1982) is an early empirical study that reports a hump-shaped growth-income path. See also Liu and Stengos (1999).

y, at the horizontal axis. The gray line depicts the relation predicted by the Solow model, where the growth rate peaks at the lowest income level and falls until it reaches the growth rate of the modern steady state

Figure 1. Hypothetical transitional growth paths



The black growth path depicts a hump-shaped relation from y^{Tr}, where the growth rate diverges from the level of the traditional steady state, g^T. It peaks at g^{max}, at an intermediate income level, y^P, and converges to g^M. The positive slope of the path between the income level, y^{Tr}, and the peak, y^P, implies that income levels diverge as growth increases with higher income. After the hump, income levels converge to the level of the leading economies along a negative slope. Countries that have reached y^{CU} have higher growth than g^M, so they catch up.

The claim of a low-level equilibrium trap is shown as a section of (black) transition curve with a negative slope below y^{Tr}. Here income is kept at a low growth rate around g^T, as gains beyond y^T result in a lower growth rate than g^T. A country caught in the trap would need a *big push* to pass the pivotal income, y^{Tr}, where it escapes persistent poverty.[7]

7. The Stalinist or Maoist models of industrialization enforced a big push by a dramatic shift from consumption to accumulation. The necessity of a big push to generate enough accumulation to catch up also follows from the Harrod model of development that led to the recipe for development aid by Chenery and Strout (1966) and perhaps Sachs (2005).

3. Data, samples, and kernel specifications

Income y is ln *gdp*, while the growth rate is $g = (gdp - gdp_{-1})/gdp_{-1}$. Initial income, y_{-1}, is termed y_{-} in the stacked data. The level of income in our sample covers an interval of about 5 log points, which represents a *gdp* difference of about 100 times. Closing this income gap within a century requires an average annual growth rate of about 7 percent – this is rare. Full transitions of poor countries typically take a couple of centuries.

Table 1. Descriptive statistics for alternative samples, 1950-2016

		All	*Basic*	*Main*	*OPEC*
Count, *N*	Observations	10,329	10,155	9,289	866
	Countries	169	169	153	16
Growth, *g*	Average	2.51	2.48	2.46	2.67
	Standard deviation	(8.17)	(6.25)	(5.94)	(8.92)
	Maximum	133.63	27.50	27.49	27.5
	Minimum	-62.94	-22.48	-22.48	-22.48
Income, *y*	Average of log *gdp*	8.48	8.48	8.44	8.94
	Standard deviation	(1.16)	(1.15)	(1.14)	(1.15)
	Maximum	12.30	12.30	11.31	12.30
	Minimum	4.90	5.02	5.02	6.72

Note: *Basic*: Truncated for outliers outside the mean 2.51 ± 25. *Basic* is divided in *Main* and *OPEC* samples, which contains the counties that are or has been OPEC members including the two new African members: Congo (Br) and Equatorial Guinea. The differences between the samples are shown on Figures 3, A1 and A11. The *All*, *Basic* and *Main* sample give almost the same kernel. Source Maddison Project Database (2018).

3.1 The Maddison cgdppc-series *1950-2016 and the four samples listed in Table 1*

We use per capita *gdp* data (multiple benchmark series) from the most recent version of the Maddison Project Database (2018). For each year in 1950-2016, the data include from 140 to 166 countries, with some data for 169 countries in total. These countries account for more than 95% of the world population. The observations for some countries start even before they became independent. For instance, observations for Sub-Saharan African countries start in 1950 when all but three countries were colonies, and observations for Yugoslavia, the USSR, and Czechoslovakia continue after 1990. We use the series for as many countries as possible, but delete overlapping observations for the three mentioned countries with their successors. This gives our ***All sample***.

Second, we truncate the observations for outliers with growth outside the mean 2.51 ± 25. This gives our ***Basic sample***. Extreme negative growth rates are apparently related to (civil) wars, the breakdown of the Soviet Union, or large changes in oil prices. Extreme positive growth rates appear to reflect the exploitation of newly discovered natural resources and mean reversion after negative shocks. Outliers account for 178 of the 10,333 observations.[8] Figure A11 in Appendix shows that alternative truncations have small effects..

We divide the *Basic* sample in the ***Main*** and the ***OPEC*** sample. Resource-rich countries often maintain a social, political, and cultural structure that is typical for low-income levels and not compatible with a modern steady state. Such countries may reach high-income levels exclusively from resource rent, without going through the Grand Transition. The OPEC group is an extreme version of resource-rich countries.

3.2 The standard kernel on the (g, y_{-})-data-pairs

All data-pairs are stacked and sorted by income. The kernel is a local polynomial smoothing process on this data string. It provides a data-driven identification of the transition path without imposing a functional form on the estimation equation. Stata requires two choices: the kernel function and the degree of the polynomial smooth.[9] The defaults are the Epanechnikov kernel and a polynomial smooth of degree zero. These choices give an optimal bandwidth. Our ***standard kernel*** keeps the first two settings, and a bandwidth of 0.35, which is close to the optimal bandwidth for the *Main* sample.

Kernel regressions come with confidence intervals that allow us to assess if theories about the form of the transition curve can be rejected. When the 95% intervals are narrow, the kernel regression line is a good representation of the data, and it gives a good identification of the different slopes over the income ranges. If countries cluster in groups, the intervals may remain large even for a high number of observations. This will be discussed below for the case of the OPEC member countries and in the Appendix.

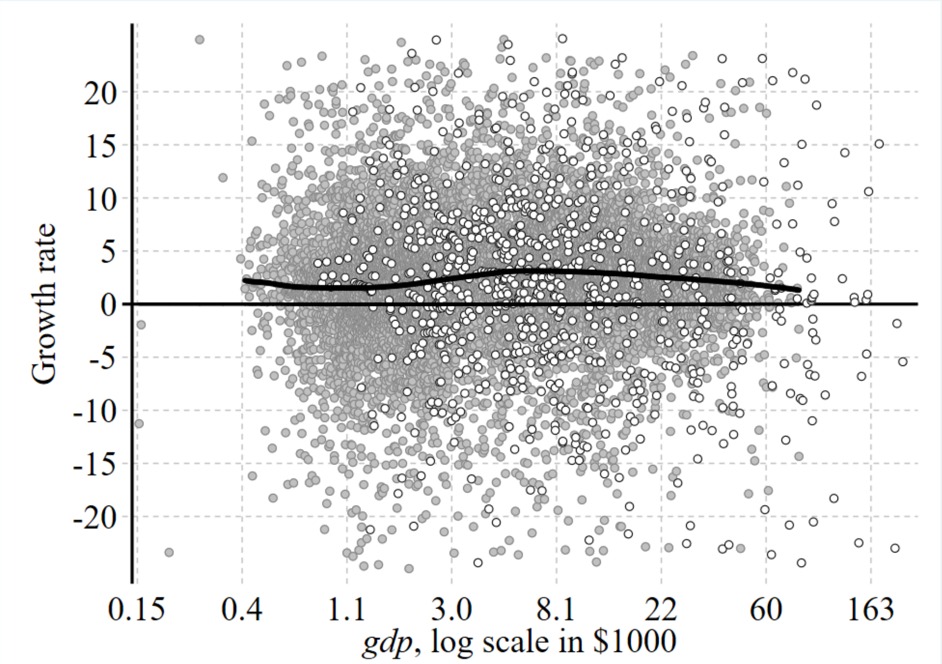
The kernel regressions we use have two related problematic features. One is that the cross-section and the time dimensions of the data are merged, which assumes that cross-country and time-series effects are similar. The second is that it is difficult to interpret multidimensional kernels. At present we only study the mono-causal relation from y to g.[10] However, we have made many robustness tests, including panel regressions with a quadratic term to catch the hump.

8. A statistical test based on the BACON algorithm, coded in Stata by Weber (2010), confirms that extreme growth rates are outliers; see also page 7-8 in the Appendix.
9. With 10,000 observations, the kernel is robust to the choice of kernel function, but it is sensitive to the bandwidth, as documented in the Appendix.
10. See Murtin and Wacziarg (2014) on the interpretation of multidimensional kernels.

4. Kernel regression estimates of the growth-income relation

Section 4.1 looks at the scatter of the *Basic* sample and the kernels in the *Main* and the *OPEC* samples. Section 4.3 looks at the path of the volatility of the growth rate

Figure 2. A scatter plot of the growth-income relation. *Basic* sample



Note: See Table 1. GDP per capita is *gdp*. White dots are *OPEC* sample. Standard kernel to *Main* sample included. The data are very thin below 0.4 and above 80 for both datasets. This means that the confidence intervals 'explode'. Therefore, the two kernels of Figures 3a and b below, use the gdp-intervals indicated on these figures.

4.1 The full scatter and two kernel

Figure 2 is the scatter of the *Basic* sample. Each dot represents one of the 10,155 observations of the (growth, income)-pair. The white dots are the *OPEC* sample, which accounts for most of the large variation of the scatter at the high-income levels – notably the majority of the negative growth rates. The wild scatter of the data points means that any transition path will explain only a small fraction of the variation. The black curve in the middle of the scatter is the kernel.

The kernel for the *Main* sample on Figure 2, as enlarged on Figure 3a is our main result. It shows a hump-shaped transition path as expected. The confidence interval is sufficiently narrow to rule out a great many possible shapes of the transition path. It is impossible to draw a straight line within the confidence intervals over the full income range. If a model that imposes a (log) linear path is estimated on these data, it will surely produce unclear results, as it is known at least since Barro and Sala-i-Martin (1992).

Figure 3a. The transition path for the *Main* sample

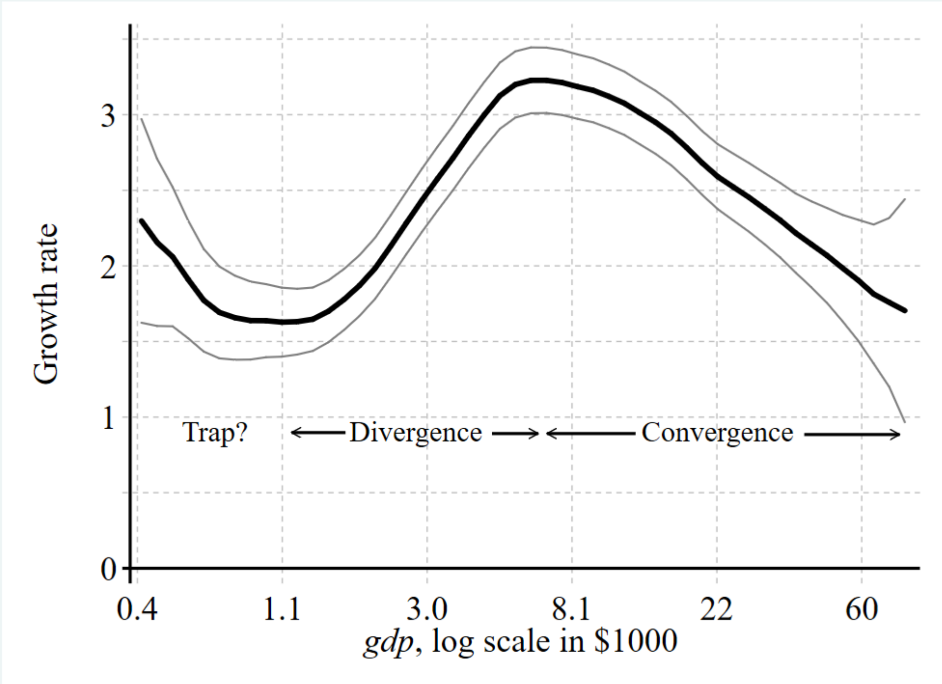
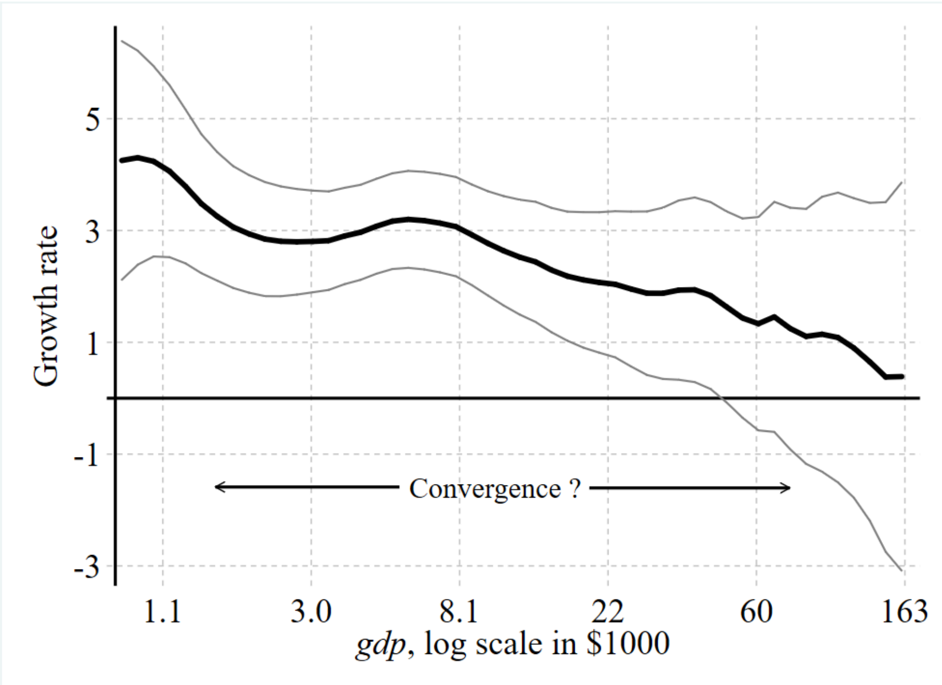


Figure 3b. The transition for the *OPEC* sample



Note: Standard kernels. The samples has a few scattered observations outside the intervals shown; see Figure 2. Note the change of scale on both axis. Figures A2 and A3 reports experiments with the bandwidth.

Figure 3b shows the kernel for the *OPEC* sample. The sample is much smaller, so that the confidence intervals are wider. The kernels on Figure 3a and 3b are significantly different at some of the range. The OPEC-kernel is almost linear with a negative slope throughout, though it is of dubious significance.

Oil sectors employ few people and use an international technology. Oil facilities are carefully fenced and heavily guarded, so they become an international enclave, with few spill-overs. The effect on the rest of the economy is limited to the tax on the resource rent that flows into the treasury, and causes public consumption. This gives a loop-sided transition, so it is no wonder that the transition curve looks different. Dutch Disease provides some of the explanation, especially if it is expanded into a political economy theory; see Paldam (2013) for a discussion.

4.2 Two dubious and one strong finding

We have looked for the low-level equilibrium trap, and the curve for the *Main* sample does have a section below $1,100, where the slope is negative as suggested by trap-theory. However, the data are thin, where the action occurs, and the confidence interval is so broad that it is consistent with a horizontal line. Also, the negative slope proved unstable in the bandwidth-experiments, see Figure A2. Figure 3a shows a clear peak at about $6,500. The experiments reported in the Appendix show that the peak exists in the *Main* sample for a broad range of bandwidths and that it is fairly robust over time and across country groups at an income range of about $3,000-$8,000.

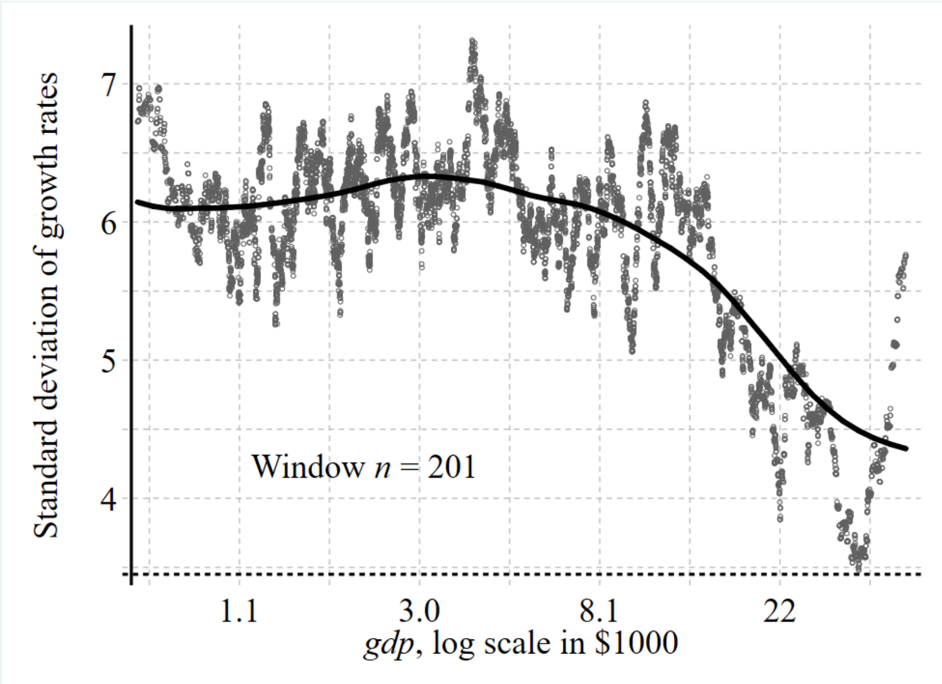
During the transition, large sectoral shifts occur, and consequently political alliances and even the political system are unstable (see Paldam and Gundlach 2018). Thus, the growth rate is unstable. In traditional society, the agricultural sector dominates, and it is susceptible to the hazards of the climate, so the growth rate is unstable here too. In the modern steady state, the structure of the economy becomes more complex, so shocks may hit one sector only. Therefore, we expect to see a fall in the volatility of the growth rate.

This is indeed the case as shown on Figure 4, which uses the sorted data set for the *Main* sample. It calculates a standard deviation on a rolling window of growth rates:

(3) $sd(g_{jn}) = sd(g_{j-\kappa}, g_{j-\kappa+1}, \dots, g_j, \dots, g_{j+\kappa-1}, g_{j+\kappa})$, where the rolling window of n observations is centered at g_j. It extends with κ observations to both sides ($n = 2\kappa + 1$).

This gives $N - n + 1$ data pairs $(sd(g_{jn}, y_j)$. The average standard deviation of the growth rate across the whole income range in the *Main* sample is 0.048 (see Table 1). Figure 4 shows that for a rolling window of $n = 201$ observations, the standard deviations scatter around the averaged level. The kernel shows the systematic movements: It is rather flat below $8,000, and then it falls to almost half. It is arbitrary to select a moving window of $n = 201$ observations for the calculation of the standard deviation of the growth rate, so we also use alternative moving windows. Figure A5 shows that the estimated kernel line is robust to alternative windows.

Figure 4. The declining variation of the growth rate with rising levels of income



Note: See Figure 3 and the text. *Main* sample and standard kernel. Figure A5 show that kernels for other windows are similar.

The declining standard deviation of the growth rate beyond income levels of about $9,000 reveals that the downward sloping part of the hump-shaped growth-income path is estimated with less variation at high-income levels than the flat and the upward sloping parts. The declining variation explains why the confidence interval for the *Main* sample in Figure 3 does not widen despite a smaller number of observations at higher levels of income.

5. Robustness

The analysis of the robustness has required the estimation of many kernels, so it is rather bulky to report. Thus, most of these experiments are found in the Appendix, with figures A1 to A11.

5.1 Bandwidths, time intervals, and windows for the std. Appendix Figures A2-A4

Figure A2 demonstrates how the bandwidth affects the estimated kernel line on Figure 3a. The kernel line becomes quite wobbly for a low bandwidth and approaches a straight line for a high bandwidth. In between, the hump-shaped path remains though the hump moves upward for high bandwidths. Figure A3 show that the same robustness is even stronger for the OPEC countries on Figure 3b.

Figure A4 shows that the path of the standard deviation of the growth rate on Figure 4 is robust to a wide range of windows. Growth regressions often use averages of intervals of 5 or 10 years. Figure A5 shows kernel regression lines for these intervals. The increasing interval size reduces the number of observations so the confidence intervals widen, but the hump-shaped form of the kernel remains.

5.2 Time periods and country groups. Appendix Figures A6 and A7

The *Main* sample is split into six decades (+1 year) on Figure A5. The hump-shape remains, with one exception: In the 1970s the oil-crisis led to low growth notably in the low income countries.

In the 1990s, 15 formerly socialist countries suddenly became 35 independent countries that experienced a shock transition to a capitalist system. Nearly all of these countries saw a large initial fall in income. Thus, the flat top of the kernel for the 90s is understandable. We have divided the European countries in two groups, where Europe W include the four overseas western countries and Israel. Europe E include the Asian countries that changed out of Socialism around 1990. Figure A6 shows the kernels for the six groups. The kernels partly reflect that these regions have different income ranges.

Europe W is dominated by high-income countries. Thus, the downward-sloping part of the transition curve is clear, while only a small part of the curve before the peak is visible. In *Europe E* the dramatic changes in the 1990 give an unclear curve, but at the high-end the curve approaches the curve for Europe W.

Asia and Pacific includes low-income countries like Nepal and high-income countries like Japan and the other high-income countries of the Far East, so here the full hump-shaped growth-income path is visible. Also, the peak is high indeed.

The kernel for *Latin America* is hump-shaped as well, but at a rather low level. *Sub-Saharan Africa* is dominated by low-income observations, showing the diverging part of the curve. The kernel line is estimated with wide confidence intervals beyond income levels of about \$5,000. Deleting the oil countries Gabon and Equatorial Guinea plus the Indian Ocean island Mauritius from the sample removes the upward sloping part of the kernel line beyond income levels of about \$6,000, thereby revealing that a mini-hump holds for an adjusted *Sub-Saharan Africa* sample as well.

By comparison, the kernel line for the *MENA* countries (Middle East and North Africa) is less influenced by the income range, but rather by the presence of many OPEC member countries, which gives a downward sloping path as shown in Figure 3b.

Table 2. Panel regressions explaining the growth rate by the log to initial income

N = 9,289	Pooled	Fixed effects for		
153 countries	OLS	Countries	Time	Both
Initial income	0.054	0.059	0.075	0.079
	(0.009)	(0.018)	(0.012)	(0.017)
Initial income squared	-0.003	-0.004	-0.004	-0.006
	(0.000)	(0.001)	(0.001)	(0.001)
R^2 within (time-series)		0.01	0.07	0.08
R^2 between (cross-country)		0.13	0.06	0.17
R^2 overall	0.01	0.00	0.07	0.02
Marginal income effects at:				
8 log points ($3,000)	0.006	-0.003	0.006	-0.010
	(0.001)	(0.002)	(0.001)	(0.003)
9 log points ($8,100)	0.000	-0.010	-0.003	-0.021
	(0.001)	(0.001)	(0.001)	(0.002)
10 log points ($22,000)	-0.006	-0.018	-0.011	-0.032
	(0.001)	(0.002)	(0.002)	(0.003)

Note: *Main* sample. Regression constant not reported, robust standard errors in parentheses.

5.3 The regression of Table 2

The reported kernel regressions are univariate and ignore the panel structure of the data. To address both concerns, we approximate the hump-shaped kernel line with panel regressions that include a quadratic income term and fixed effects. We check if the marginal income effects change from positive to negative with rising levels of income as predicted. The statistically significant coefficients are positive to income and negative to squared income in all regressions. As expected, the explanatory power is low.

Column (1) of Table 2 gives the Pooled OLS results, which serve as a point of reference. The marginal income effects change as predicted by the kernel line: positive at low-income levels and negative at high-income levels, with all estimated effects statistically significantly different from zero. The negative coefficient of -0.006 at the high income level implies a slow rate of convergence of about 0.6%.

The results change with the introduction of country-fixed effects in column (2). Eliminating the cross-country variation from the sample is like assuming that all countries are the same except for their income level, so it is almost by default that negative marginal income effects are estimated at all three income levels. At the low-income level, the estimated coefficient is statistically not different from zero. At the high-income level, the negative coefficient of -0.018 implies a convergence rate of about 2%, which is in line with results reported in the conditional convergence literature noted above.

Column (3) reports results for the inclusion of time-fixed effects, which eliminates the effects of common shocks from the sample but retains the cross-country variation. Like the pooled OLS, this specification produces a reasonable approximation of the kernel line: the marginal effects are estimated for low and high-income levels. The implied convergence rate at the high-income level is about 1% and not much larger than the implicit divergence rate at the low-income level, which implies a net convergence rate substantially below 1%. Finally, column (4) reports results for the inclusion of both country- and time-fixed effects. Not surprisingly, the marginal effects are much like the marginal effects estimated with country-fixed effects only.

Overall, the results in Table 2 confirm the hump-shaped kernel line of Figure 3a if the cross-country variation is maintained (POLS, TFE), and they confirm the results of the conditional convergence literature once it is eliminated (CFE, 2FE). Not controlling for obvious cross-country differences except for the level of income will necessarily produce an omitted variables bias, but eliminating all cross-country variation may be too much of a good thing, especially when assessing a potential pattern of long-run growth and development. After all, the long-run information appears to be in the cross-country variation of income levels, not in the within-country variation of income over time.[11]

The Grand Transition from the traditional to the modern steady state is a hypothesis that relies on cross-country and on time series evidence. Treating the cross-country variation as a source of omitted variables bias must lead to a rejection of the Grand Transition hypothesis by default, because the within variation of growth rates in 1950-2010 does not suffice to capture the transition from a static to a modern steady state. Maintaining the cross-country variation helps to identify a hump-shaped growth-income path with both kernel and panel regressions. This is not to deny that the level of income only explains a tiny fraction of the observed variation in growth rates across countries and over time, but ignoring the Grand Transition pattern means missing a signal in the noise.

6. Understanding the hump-shaped growth-income relation

The hump-shaped relation can be understood at several levels. Section 6.1 looks at a couple of intuitive levels, while section 6.2 shows that it can be simulated by a two-sector model.

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11. Hall and Jones (1999) use this argument to motivate their cross-country regressions on the effect of institutions on long-run economic performance. Along the same lines, Frankel and Romer (1999) use cross-country regressions in *levels* to estimate the effect of trade on (long-run) growth. The combination of persistent country characteristics and non-persistent within-country growth rates, which has been emphasized by Easterly *et al*. (1993), also speaks against eliminating *all* cross-country variation from the sample, because otherwise nothing but regression to the mean may be left.

6.1 The intuition

On the deepest level we see the transition as a change between two steady states. In each of them all relations are stable – i.e., all growth is technological. Hence, the two curves of Figure 5b. The gray curve is the mechanical conversion of Figure 5a, while the black curve adds technological progress. When the steady states are far apart, a transition has to be a long period of changes. Here growth must be relatively high – not every year, but in average.

Since many level variables in the economy have transitions that will affect the aggregated growth rate of income (a first difference-variable), it is no wonder that Figure 3a looks like Figure 5b. The change of perspective from Figure 5a to figure 5b increases noise in the relation dramatically. While the correlations between the level of a typical transition variable and income is perhaps 0.6, it will necessarily be much lower between the first difference and income. The relation we analyze is 'born' as a relation between the growth rate and (initial) income, and it does have a low fit, but it is statistically significant when measured appropriately.

Figure 5a. The transition in levels

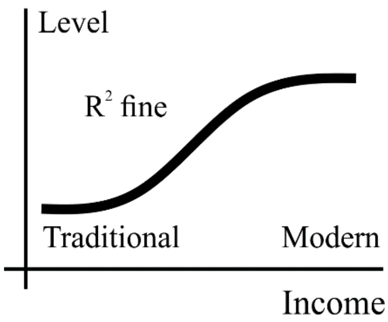
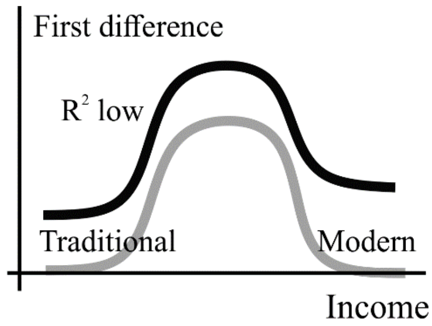


Figure 5b. The transition in first differences



On a more concrete level we can understand the hump by a model with two sectors, a traditional and a modern, that differ by technology, and hence productivity. Growth has three sources. One is the slow growth within the traditional sector, the second is the growth within the modern sector, which is determined by international technological progress. The two internal growth rates give a sum weighted by the shares of the sectors, so it gives 1-3% growth. However, to this should be added the third source – sometimes termed the *Petty-effect* – that results from structural change, namely the transfer of capacity (notably labor) from the low productivity, overpopulated, traditional sector to the high productivity modern sector. If Δs is the share of capacity moving and the productivity gap is p, then the growth gain is Δsp. This is potentially a large number. If $p \approx 5$ and $\Delta s \approx 1\%$ it gives a potential extra growth of 5%, so that we end up with about 7% growth. This is, of course, if everything goes well – but it rarely does, as modeled by the many versions of the Harris-Todaro model (Harris and Todaro 1970). Thus, countries will typically grow less than the potential 7%.

The size of the Petty-effect depends upon two variables. The productivity gap *p* falls as labor leaves the traditional sector, which includes much hidden unemployment, and the speed of structural change Δs depends upon the absorptive capacity for resources in the modern sector, which in turn depends on the demand for the products of the sector. At the early stage of the transition, the modern sector consists of just a few islands of modernity that cannot absorb very much, but once it becomes substantial, it can absorb a great deal. It is easy to imagine an almost endless number of possibilities, giving more or less successful development processes. This tally well with (i) the existence of a growth hump, but also (ii) with a hump that is much smaller than the potential, and (iii) with the large variability in the growth rate until quite late in the transition.

6.2 A simulation of the transition in the growth rate

The reader will have noted that most of the cited literature on the two-sector model of development is quite old. Lucas (2009) reintroduced the two-sector model in a version that refers to modern growth theory and to empirical evidence on cross-country patterns of catching up and convergence. The Lucas model has a traditional farm sector with low capacity to absorb international technology and a modern city sector, which is the absorption hub for such technology. Both sectors produce a single output good that adds up to GDP.

All countries are represented as one-factor economies, where GDP per person is proportional to the single factor input, *K*, which is called capital and may be interpreted as a broad concept that also includes human capital. In the leading economy, the stock of capital and per capita income are assumed to grow at the exogenous steady state growth rate, γ, which is given by

(4) $$K(t) = K(0)e^{\gamma t} \quad \Rightarrow \quad \gamma = \dot{K} / K = g_{\gamma}$$

In any other (follower) economy, denoted by small letters, the stock of capital is assumed to evolve according to

(5) $$dk / dt = \dot{k} = \gamma k^{1-\psi} K^{\psi}$$

with ψ as an externality linked to the accumulation of capital, where higher values of ψ indicate a more productive environment. Equation (6) implies that the growth equation of a follower economy is given by

(6) $$g_y = \dot{k}/k = \gamma(K/k)^{\psi}$$

Hence, the growth rate of any follower economy positively depends on the proportional income distance to the leading economy (*K*/*k*) and the size of the accumulation externality, ψ. For any positive value of ψ, equation (7) implies a log-linear relation between the growth rate and the initial capital of a follower economy. A larger capital gap relative to the leader implies a higher growth rate of the follower, thereby implying a process of unconditional catching up that is not supported by the data, as discussed in section 4.

To derive an equation that generates a hump-shaped growth-income relation, Lucas (2009) introduces two further externalities, which can only be motivated with a two-sector structure of the model. A *knowledge accumulation externality*, ζ ,is assumed to result from agglomeration, such that knowledge accumulates faster with increasing size of the modern city sector. A *sectoral productivity externality*, ξ, is assumed to increase the productivity of the traditional farm sector in proportion to the (rising) productivity of the modern city sector. The two additional externalities change the one-sector growth equation (6) to a two-sector growth equation:

(7) $$g_y = \gamma\left[1-\left(\alpha A/k^{1-\xi}\right)^{1/(1-\alpha)}\right]^{\zeta}\left(K/k\right)^{\psi},$$

where the whole term in brackets represents the employment share of the city sector, α is labor's share in farm production and *A* is a constant that includes farm land per person.[12]

The growth path described by equation (7) has a number of features. For instance, a positive growth rate can only result if the term in brackets (the added knowledge accumulation externality) is positive, which is guaranteed by assuming a minimum amount of *k* that is large enough to allow for any growth at all. If *k* remains below the critical level, the predicted growth rate approaches 0 with $k \to 0$, all else constant. If *k* is increased above the critical level, the growth rate will reach a maximum where the growth drag of a large farm sector is overcompensated by the capital gap. With $k \to \infty$, the term in brackets and the capital gap term (in parentheses) will both approach 1, which implies that the growth rate of the follower economy, g_y, will gradually approach the growth rate of the leading economy, γ, in the long run.

12. For details of the derivation of the two-sector growth equation, see Lucas (2009).

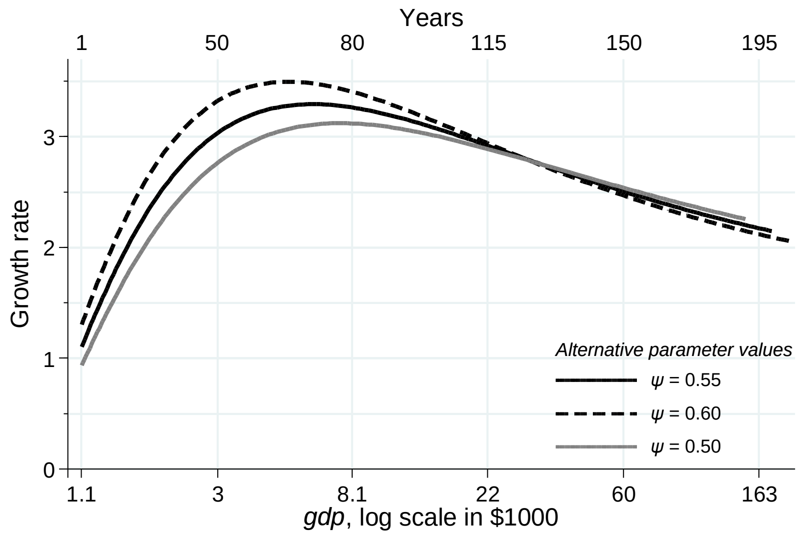
Hence, equation (7) predicts a hump-shaped transitional growth path: a low growth rate at low levels of capital, a rising growth rate after a critical level of capital has been reached, and a gradually declining growth rate over the subsequent process of catching up with the leading economy. The growth advantage of extreme poverty implied by the one-sector model of equation (5) is eliminated by the introduction of a two-sector structure, conditional on the assumed growth drag of the traditional agricultural sector. What remains to be seen is whether reasonable calibrations of equation (7) generate transitional growth-income paths like the ones identified by the kernel regressions.. Taking Figure 3a as the point of reference, the simulated growth path should start with a growth rate below 2% at an income level of about \$1,100, peak at a growth rate between 3% and 3.5% at an income range between \$3,000 and \$20,000, and converge to a growth rate near 2% at income levels beyond \$50;000.

The calibration of equation (7) demands initial values for capital (proxied by income) and the employment share of agriculture together with a set of parameter values for γ, , ψ, ξ, ζ, α, and A For the simulated growth path, we consider an economy with an income level of \$1,100 as the starting point. This is about the lowest income level where the kernel line of Figure 3a is supported by reasonable confidence intervals. We further assume that our model economy is far behind the leading economy described by equation (4), with a relative income of 5.5%. The constant growth rate of the leading economy is set to γ = 1.5%; the sectoral productivity externality is set to $\xi = 0.8$; labor's share in farm production is set to $\alpha = 0.6$. For given ξ and α, the implied parameterization of A can be backed out for a given value of k_0, which can itself be derived from the equations of the model for an initial income level. With $y_0 = \$1{,}100$, it follows that $k_0 = \$733$ and $A = 5.56$. The initial share of employment in the farm sector is set to $x = 75\%$.

The parameterization of the capital accumulation externality, ψ, is confined by the range [0,1], because a negative value would imply that there is no catching up with the leading country at all, while a value higher than 1 would imply that the poor country would eventually overtake the rich country. We use alternative values for ψ, with 0.55 as our preferred parameterization. The value of the knowledge accumulation externality is more difficult to pin down, because any positive value may be used.[13] Conditional on the previous settings, we select $\zeta = 1.4$ as our preferred parameterization.

13. For $\zeta = 0$, equation (10) equals equation (5); Lucas (2009) uses $\zeta = 1$ as his preferred parameterization.

Figure 5. Simulated growth-income paths

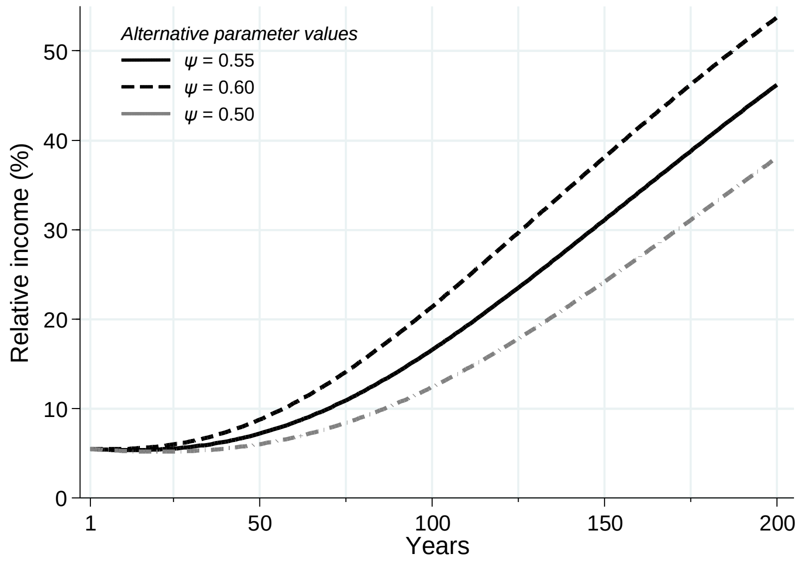


Note: See text for other parameter values. The timeline refers to the preferred specification with $\psi = 0.55$ and $\varsigma = 1.4$ (in black).

Figure 5 shows simulated growth-income paths of a hypothetical poor economy over $T = 200$ years (periods) for alternative parameterizations of the capital gap externality. The simulated growth path resembles the empirical growth path in Figure 3a quite well. Growth starts at a rate below 2% and rises above 3.% at an income level beyond $3,000 for the preferred specification (black line). After the peak, the simulated growth rate falls persistently toward 2%, though not as fast as suggested by the empirical growth rate in Figure 3a. The simulated growth path implies that the model economy would need more than 50 years to reach its growth peak. Figure A8 gives about the same results for alternative knowledge accumulation externalities in combination with the preferred capital gap externality.

Figure 6 shows a translation of the simulated growth-income paths into relative income levels. Starting at a relative income level of 5.5%, the model economy would not improve its relative income position by much for the first 50 years, i.e. before it reaches its growth peak. Even after 150 years (preferred specification), the model economy would have reached only about one third of the income level of the leading economy. The simulated process of catching up implies a very low rate of convergence, far below the "iron law" rate of (conditional) convergence in the vicinity of 2% (Barro 2015), which can be motivated by a one-sector growth model. With a convergence rate of 2%, halfway to the steady state would be reached after 35 years, and two thirds of the steady state would be reached after 55 years. Our simulations suggest that an average country would reach halfway to the steady state only after more than 200 years. To put this number into perspective, a halfway time of 200 years implies a rate of convergence of 0.35%, which is in line with the low convergence rate derived from column (3) of Table 2.

Figure 6. Simulated growth-income paths by institutional quality over time



Note: See Figure 5.

Hence, the main message of the simulations appears to be that economic catching up is an extremely slow process under average conditions. Observed growth miracles in some countries in Europe and East Asia after the Second World War are probably just that: exceptions from the general pattern of slow long-run development that are due to lucky circumstances. The common growth pattern that appears to underlie the cross-country panel data is so weak relative to all other positive and negative shocks that it cannot be identified with the standard tools of regression analysis, but ignoring a common pattern, even if it is weak, appears to be a mistake.

7. Conclusions

Recent empirical research on a general pattern of growth and development across countries and over time has mainly concluded that such a pattern does not exist: if anything, countries are said to converge to individual steady states rather than to a common steady state. Reconsidering the empirical evidence with kernel regressions reveals a common hump-shaped transition path, where growth rates peak at an intermediate income level.

We find that the *poorest countries* have low and unstable but positive growth rates. The static Malthusian steady state is all but gone, though there are a few poor countries with income levels that are lower in 2016 than in 1950.[14] The *richest countries* (excluding OPEC members) are found to have an average annual growth rate similar to the one of the poorest countries, but with a variation that is only about half as large.

Middle-income countries are found to have an average annual growth rate that is about one percentage point higher than the growth rate of the poorest and the richest countries. Excess growth of one percentage point accumulates to a difference in levels of one log income point over a century. This means that middle-income countries do catch up with the high-income countries, but this also means that reaching income convergence will be a matter of centuries rather than decades.

Most of the enormous variation of observed growth rates remains unexplained by our results, but we have found a clear transition path in the data. Such a transition path can be simulated on the basis of a two-sector model, which avoids the straightjacket of a log-linear growth-income relation implied by the one-sector workhorse model of growth empirics. The long-run information about economic development is held to be in the cross-country variation of income levels, not in the (short-run) growth rates. When employing the usual dose of fixed effects, one risks missing the signal in the noise, namely a hump-shaped growth-income path that appears to hold as a general pattern of long-run development.

14. This group includes Central African Republic, Liberia, Madagascar, Niger, Congo (Kinshasa), and Haiti.

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