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Preliminary

A new look at the growth-income relation

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Abstract:

The paper considers all pairs of growth and initial income in the updated Maddison data from 1950 to 2010. The analysis confirms the well-known fact that the growth-income relation has low explanatory power, but when it is analyzed by kernel regression techniques, a highly significant hump-shaped relation appears. It gives the long-run dynamics of the incomes of the countries of the world. Further, a moving measure of variation is developed for the growth rate. It is shown that the variation in growth rates has a large fall from middle income to high income. Also simulations with a two-sector model are shown to generate a similar looking hump-shape.

Keywords: Non-linear growth, convergence, low level equilibrium trap

Jel: O4, C4

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1. The growth-income relation

Few relations in economics have been researched as much as the growth-income relation, $g_i = G(y_{i(-)})$. It considers growth, g, as a function of initial income, $y_{(-)}$, defined as the logarithm to GDP per capita, in fixed PPP prices. The G-curve is often illustrated by a growth-income diagram, with growth at the vertical axis and initial income at the horizontal axis. In large data-sets the (g_i, $y_{i(-)}$)-pairs scatter wildly, but behind the scatter is the 'underlying' G-curve that shows the transition from the traditional to the modern steady state.

The paper studies the G-curve by kernel regressions.[3] This technique has two advantages: (a1) It tells the 'stylized' story about the path in the data independently of the functional form. (a2) It works with stacked and sorted data, so that all other country differences than income are randomized. Therefore, we take the (g_i, $y_{i(-)}$)-kernel curve to be an estimate of the G-curve, which is long-run development path of the average country.

These advantages come at two disadvantages. (d1) The study merges data across time and countries. This assumes *equivalence*: Long-run and cross-country data tell the same story at the macro level. (d2) The kernel-technique is a semi-graphical univariate technique. Obviously the world has more dimensions. However, we consider a near-consistent set of N = 8,954 pairs of observations. This is enough data so that it could be broken up in many ways, giving some insight in other dimensions.

The kernel has a robust path, which proves highly non-linear. It does answer some much discussed questions, of which we look at three: (i) It demonstrates that the G-curve is hump-shaped and not linear as assumed by most growth theorists. (ii) It does not find a low level equilibrium trap. (iii) It shows that the West is an open convergence club.

The content of the rest of the paper is: Section 2 considers the theories. Section 3 describes the data and argues that they are rather representative. Section 4 looks at the pattern in the data. Section 5 considers the development in the variation in the data. Section 5 simulates the path founds using a two-sector model. Finally, section 7 concludes.

This paper is written on the basis of a longer paper, which documents many additional results, see Paldam and Gundlach (2015).

3. Readers not familiar with the technique may think of it as a moving MA-process with a constant bandwidth (bw) using a smoothing expression (kernel). The program varies two parameters: the bw and the kernel. It is robust to the kernel, so we use the Epanechnikov kernel recommended by Stata. The bw chosen is discussed.

2. Some theory

The theory of economic growth is a large field, covered by numerous textbooks, including the massive handbook of Aghion and Durlauf (2005, 2013). Thus, a few notes on a couple of central themes of relevance will suffice. For ease of presentation we use the World Bank terminology LICs, MICs, and HICs that refers to Low, Middle, and High Income Countries.

Growth theory tends to be easier to handle in a log-linear so we define income, y, as the logarithm to *gdp*, which is GDP per capita in fixed PPP prices.

Section 2.1 deals with the standard one sector steady state model, while section 2.2 turns to the observation that the data holds two basic steady states – the traditional and the modern – that are connected by the Grand Transition, which is no steady state. Section 2.3 translates some theoretical propositions to observable properties on the empirical *G*-curve. Finally section 2.4 suggests that the *G*-curve looks different in very resource rich countries.

2.1 The Solow model and growth regressions

The discussion of the *G*-curve often starts from empirical observations of long-run time series for income, y. Most are from western countries, and they normally look amazingly linear – rarely with more than one kink per century. This is often taken to represent a Solow model growing along a steady state path. From this theoretical backbone a large wave of growth regression was started by Barro (1991) and Barro and Sala-i-Martin (1995, 2004). It demonstrates how the Solow model leads to a basic linear estimation model in two versions:

(1) $g_{njt} = \alpha_1 + \beta_1 y_{jt\text{-}1} + \varepsilon_{1jt}$, the ε is the noise term

(2) $g_{njt} = \alpha_2 + \beta_2 y_{jt\text{-}1} + [\gamma_1 z_{1jt} + \ldots + \gamma_k z_{kjt}] + \varepsilon_{2jt}$, the z's are k controls

Indices j and t are for countries and time, while n indicates an average over n years.

Equation (1) looks for *absolute* convergence that occurs if $\beta_1 < 0$. The standard result is that $\beta_1 > 0$, but statistically insignificant, so convergence is rejected. Regression (1) in Table 2 below report an estimate of (1) for N = 8,872.

Equation (2) looks for *conditional* convergence that occurs if $\beta_2 < 0$, where the z-set controls for country differences. Here β_2 turns significantly negative for reasonable and robust z-sets. It already happens when the z-set is fixed effects for countries, as shown be regression (2) in Table 2. Thus, countries would converge, if they were the same. It is almost a tautology, and it is surely not a statement about the real world. However, it is fairly easy to find more interesting control-sets making β_2 negative, as already reported in Barro (1991).

These equations have been estimated by many estimators, and on many data sets, both in pure cross-country versions, in panel versions, and with different lags between the variables. About 400 z-variables have been tried in many combinations, etc.[4]

2.2 The traditional and the modern steady state – and the Grand Transition

Researchers in economic history and development have – at least since Rostow (1960) – pointed out that two basic steady states exist: the traditional and the modern. This is confirmed by the large efforts of data collections put together by Maddison (2001, 2003).

Throughout history the most common has been the traditional steady state that produces with an almost stable traditional technology. It gives low income and little growth. It has often been claimed that poor countries is kept in that state by a poverty trap.

The modern steady state of HICs started in a few Western countries more than two centuries ago. Gradually it been joined by other countries.[5] It produces by a dynamic, modern technology, which gives a much higher income. Since Baumol (1986) it has been known that the HICs converge to much the same income, as modern technology is increasingly international. This is termed club convergence, and it sometimes discussed if the club has an open or closed membership.

Since the modern steady state emerged a large gap has grown between the countries in the two steady states. A change between two steady states is a transition, so the change between the two basic steady states is termed the *Grand Transition*. It is *not* a steady state as all relations in the society change. As the group of HICs keeps growing there must be countries that grow faster during the transition than later. This suggests that the G-curve is hump-shaped, as originally proposed by Rostow (1960).

2.3 What can we learn from estimates of the G-curve?

The estimated G-curve kernels are surrounded by a 95% confidence interval. If the countries are in very different groups or equivalence is rejected, we should find large confidence intervals which allow qualitatively different G-curves to exist. However, we reach narrow confidence intervals that strongly reduce the class of possible G-curves. The G-curve is the

4. It is only possible to include a handful or two of the 400 variables so billions of choices are possible. Each of these gives an estimate of β_2, resulting in a large range of possible estimates. See chapter 12.5 of Barro and Sala-i-Martin (2004) for a large-scale attempt to identify robust explanatory variables.
5. Various counts of HICs exist: OECD started in 1961 with 19 members; it is 35 today. The World Bank designated 28 countries and 6 microstates HICs in 1987; today it is 50 countries and 29 microstates. Data are often missing for microstates, so that it is a bit random, when they are on the list.

thick gray curve on Figures 1 and 2.

If it is hump-shaped over most of its income range, the path should look as drawn on Figure 1. One horizontal line (Lh) and three vertical lines (L1-L3) are drawn on the figure:

Lh: Horizontal line at g_h, which is the growth of the countries at the high end, y_3.

L1: Intersects the income axis at y_1, where growth is also g_h. The *G*-curve is above Lh in the interval from L1 to L3. All countries in that interval catch up.

L2: Goes through the peak of the hump. The slope of the G-curve is positive to the left of L2, giving divergence. The slope to the right of L2 is negative, giving convergence.

L3: Is the line through y_3, which is the high end, where countries grow at g_h.

Figure 1. A hump-shape

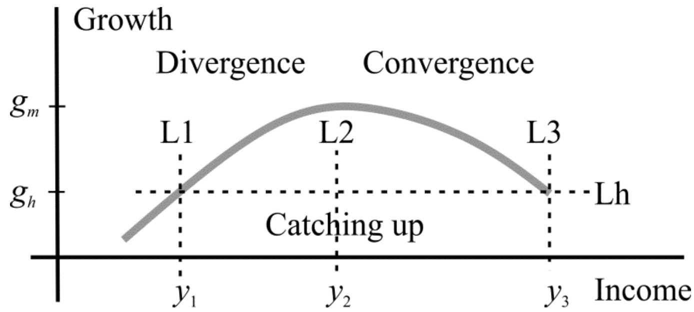


Figure 2a. A low-level equilibrium trap

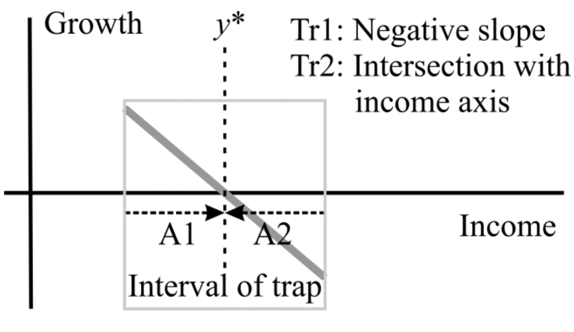
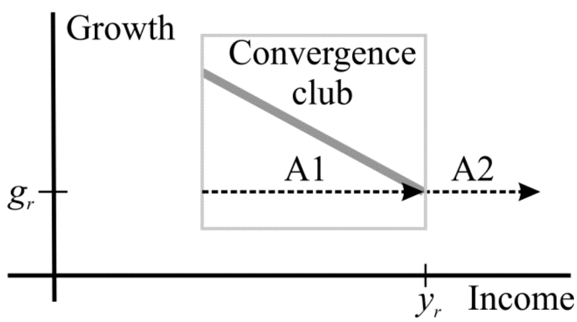


Figure 2b. A convergence club



The *G*-curve also allows us to look for growth traps and convergence clubs as depicted on Figures 2a and b. Both figures consider a limited income interval indicated by the gray frame on the two graphs. They may also cover a group of countries only. Both Figures 2 consider a section of the *G*-curve where the slope is negative, but on Figure 2a it intersects the income axis giving a local equilibrium at y^* where g^* is zero. On Figure 2b the top counties keep growing, at g_r, but the countries in the box converge. This corresponds to section of the *G*-

curve to the right of Line 2 on Figure 1, where $g_r = g_h$.

One way to understand the hump-shape is by a two sectors model with a traditional and a modern sector as demonstrated in section 5.

However, even when the two-sector model may catch an important aspect of the Grand Transition, it is a much more comprehensive process: It includes a large fall of corruption, a democratization, a large fall in religiosity, etc. These changes are an effect of growth, but they may also contribute to growth, and they may not be smooth, so they may generate political instability that affects investment. [6]

The transition started 200 years ago in a group of 'old' western countries. The transition was a gradual process, so it is easier to describe as a log-linear process, where technical progress was randomly distributed throughout the economy. They certainly had tensions between the sectors, but they were (much) smaller than the ones we see in the MICs today. This is a qualification to equivalence.

2.4 A division: The Main and the OPEC groups of countries

We have divided the countries in the five standard regional groups (West, Latin America, Sub-Saharan Africa, MENA and South and East Asia) and have found that most divisions give much the same *G*-curve though data for a region may cover only some of the income spectrum and hence miss a section *G*-curve. However, one division matters:

All countries earn some resource rent. It is typically 1 to 2% of GDP only, but some countries have abundant resources and resource rent becomes a major part of GDP. Such countries may reach HIC income levels exclusively from resource rent, without going through the process of the Grand Transition. For long they remain LICs in the structure of society and its institutions, but gradually they start to change structure, to conform to income.

The most extreme version of this case of development is the oil countries as an oil sector is a small international enclave in the country, employing few people using a foreign technology. Such sectors are normally carefully fenced and heavily guarded so they are indeed an enclave. Their effect on the rest of the economy is the tax on the resource rent that flows into the treasury. We use OPEC membership as our proxy for this case. Thus, the data is divided in the Main (with capital M) group of 8,377+ observations and a small OPEC group with 577 observations. The G-curves for the two cases are different in the two cases.

6. The reader will think of the agricultural and the demographic transitions, but we have shown that there is also a democratic transition (see Gundlach and Paldam 2009b and Paldam and Gundlach 2012 and 2016), a religious transition (see Paldam and Gundlach 2013), a transition of corruption (Gundlach and Paldam 2009a), and a transition in the preferences for capitalism/socialism (Bjørnskov and Paldam 2010).

3. The data and some linear regressions

Section 3.1 looks at the data, while section 3.1 shows some standard linear regressions.

3.1 The data

The data used is from the Maddison-project, where the annual data starts in 1800 and (p.t.) ends in 2010.[7] They include data for the present countries as soon as possible. Data for Sub-Saharan African starts in 1950, where all but three counties were colonies. Data for the successor states of Yugoslavia starts in 1952, while data for the successor states of the USSR starts in 1990, and so they does for the successor states of Czechoslovakia. In these cases the data for the 'old' country is stopped, when the new data starts.[8]

The full Maddison data covers 12,786 data-pairs. During the 19th Century 74% of the observations are from the old west; then some Latin American countries join the sample, but before 1950 the data lacks representability. From 1950 the sample holds at least 144 countries with more than 95% of the world population. The present study only covers this sample, where N = 8,954.

Income is in (natural) logarithms to 1990 international Geary-Khamis dollars. They cover the interval from 5.3 to 10.5. This is 5.2 ***lps*** (logarithmic points) which is a span of 180 times in the *gdp*. To close such a gap in a century means that an excess growth of 5.3% above the one of the rich countries. This has happened, but rarely.

Growth g is growth of *gdp*. As ln is the natural logarithm, the increase of one income unit is an increase of $e \approx 2.72$ times in the *gdp*. The income scale on Figures 4 to 7 has a range of 5 income units, so the *gdp* range covers $e^5 \approx 148$ times. Some observations are missing due to countries that came into existence after 1960.

Table 1. Some descriptive statistics

	N	Average growth	Std dev.
Main Group	8,377	2.12	5.67
OPEC Group	577	0.84	9.04
All	8,954	2.03	5.95

7 . References are Maddison (2001, 2003), Bolt and van Zandern (2013) and Maddison Project.
8. The data comes from a (g_{jt}, y_{jt-1},)-panel, where j and t are indices for country and time. The data is stacked and sorted to the (g_i, $y_{i(-)}$)-data set where i is an index for the income order. Time only appears as (-) indicating that each data pair is from the same country and the two time periods t and t-1. In the literature g_i is often an average g_{ni}, where $g_{njt} = (g_{jt} + g_{jt+1} + ... + g_{jt+n-1})/n$. See the appendix on definitions.

Table 2. Explaining growth by income and fixed effects for all data

Reg.	Income	Constant	Fixed Effects	R^2	N
(1)	0.135 (2.4)	0.965 (2.1)	none	0.0006	8,954
(2)	-0.846 (7.3)	9.992 (9.2)	countries	0.0621	8,954
(3)	0.252 (4.4)	0.953 (1.7)	time	0.0300	8,954
(4)	-0.614 (4.6)	8.977 (6.9)	both	0.1311	8,954

Note: Parenthesis holds t-ratios. The constant in the FE-estimates is reached by deleting The 29 of the 163 countries, where $-0.2 < t < 0.2$.

The fixed effects sets, are made from two complete sets of dummies for the main group, FEc and FEt, with dimensions (162 *x* 8,954) and (59 *x* 8,954) respectively. In FEc with cell fec(i, j) = 1 for all observations for country i and 0 otherwise. In FEt with cell fet(i, j) =1 for all observations for year j and 0 otherwise. When FEc is used the 10 richest Western countries are deleted to give the constant, and when FEt is used the last year 2010 is deleted to give the constant:

The results reported in Table 2 are very standard. The estimate of β_1 (for absolute convergence) in equation (1) is positive, but with fixed effects for countries included it turns negative and much more significant. Fixed effects for time matters less.

4. The Grand Transition: A hump-shaped relation

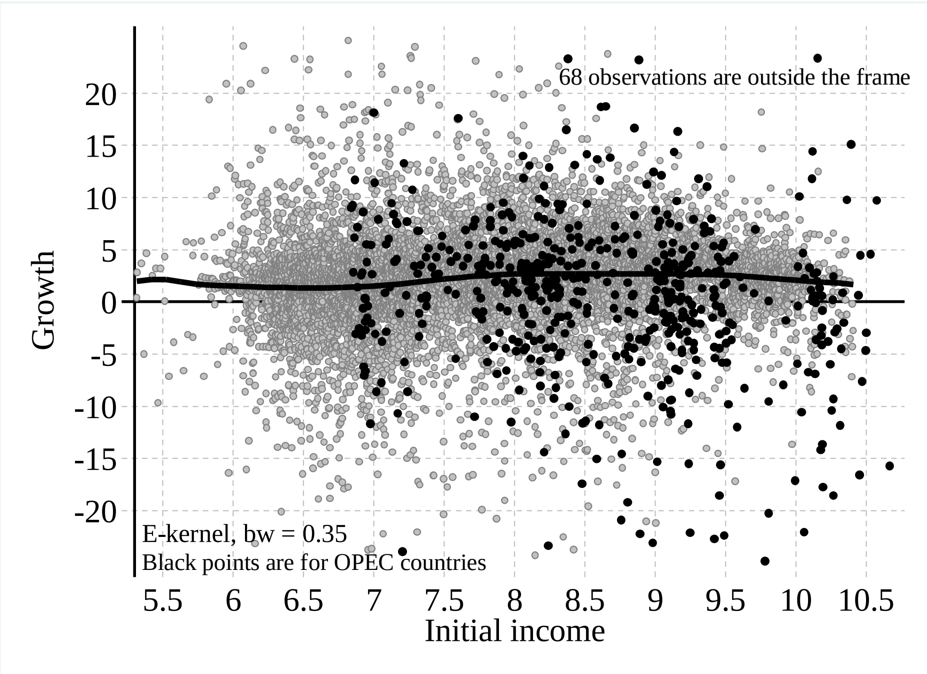
Section 4.1 gives the point-scatter of the data, while section 4.2 considers the Grand Transition in the Main group of countries. Section 4.3 looks at the OPEC countries that became wealthy without a transition. Finally, section 4.4 looks at the cross-country variation.

4.1 The scatter of the 8,872 data points

The data-pairs are shown as a scatter diagram in Figure 3. The points scatter very much, so it is difficult to see any pattern. The wild scatter means that any simple pattern, such as the one discussed, will explain a modest part of the variation only.

Figure 3 is provided with a kernel regression using the Epanechnikov kernel, with a bw (bandwidth) of 0.35. Most of the variation at the high end – notably the negative observations – is due to the oil-rich countries, so the 563 observations for OPEC members are singled out and omitted in the estimate of the kernel.

Figure 3. The scatter of all N = 8,874 from 1950 to 2010



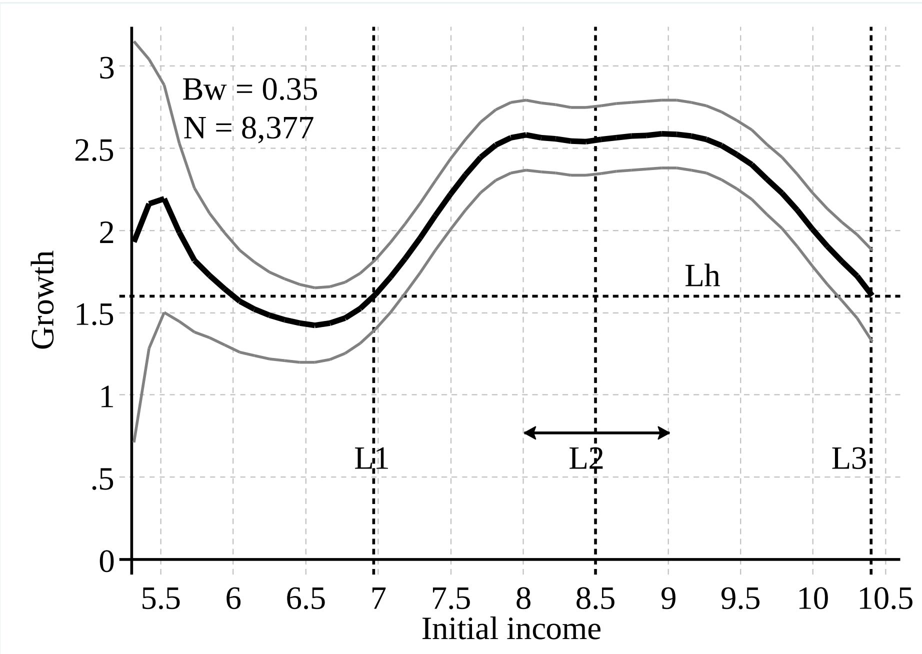
4.2 The hump-shaped Grand Transition in the Main group

Figure 4 shows how the kernel curve looks when the scatter points are suppressed so that the vertical axis can be enlarged. The 95% confidence intervals are added. Figure 3 showed that the kernel is supported by few points at both ends, but it looks as if the scatter narrows at the high end when the OPEC countries are omitted. The kernel curve on Figure 4 has the typical hump-shape found in large growth-income data sets. It is obvious that no linear curve can be drawn within the confidence interval. The richest countries at the extreme right side grow at about 1.6%. The kernel has two important properties marked by the lines from Figure 1:

L1 divides the countries, to the left and the right of the hump. Countries to the right of L1 are growing faster than 1.6%, so they catch up with the rich countries, while countries to the left of L1 does not catch up, but they do grow and will thus cross L1 and start to catch up.

This seems rather positive, but the catch-up times implied by the figure are very long. Consider a poor country at the start of the hump and a rich country at the end of the hump. They differ by app 3 *y*-points, which is a *gdp* gap of 20 times. If the poor country grows by 1% more than the rich, it takes about 300 years to catch up.

Figure 4. Kernel of the growth-income relation for the Main group



Note: 'Bw' is bandwidth and the two gray lines show the 95% confidence interval. The lines are from Figure 1. Paldam and Gundlach (2015) give estimates for a range of periods and bandwidths. The hump-shaped form is rather robust, but it only explains a small fraction of the variation; see Table 2. When the OPEC points are included in the estimate of the kernel, it is indistinguishable in most of the range, but it falls more at the high end and becomes negative, see Paldam and Gundlach (2015).

Table 3. Linear regressions explaining the growth rate

Country group	Figure	Income	Constant	R^2	*N*
		Divergence for income below hump			
Main group $6 < y \leq 8$	4	**0.855** (5.0)	**-4.176** (-3.5)	0.005	4,624
		Convergence for income above hump			
Main group $y > 8.5$	4	**-0.470** (-2.5)	**6.696** (3.8)	0.002	2,483
		Convergence for all observations			
OPEC group	5	**-1.89** (-4.7)	**17.12** (4.9)	0.035	577

Note: See Table 1.

L2 divides the countries by the sign of the slope on the kernel. It is not clear if the top is at 8 or 9 as indicated with arrow. The curve shows the well-known property of divergence (where $\beta > 0$) to the left of L2, and convergence (where $\beta < 0$) to the right of L2. Estimates of two straight lines with the said slopes are given in Table 2.

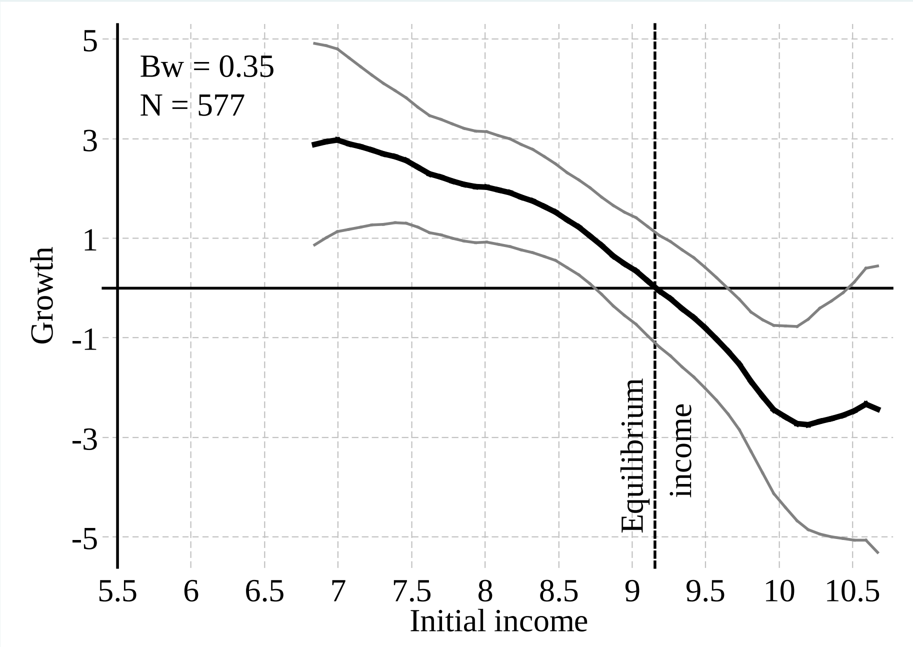
In development theory it is frequently claimed that a low-level equilibrium trap exists, so that countries that start to grow fall back to the previous income level as shown on Figure 2a; see Malthus (1803) and Azariadis and Stachursky (2005). From Figure 2a we know that such a trap a certain interval should exist where the *G*-curve have two properties: (Tr1) it intersects the horizontal axis, and (Tr2) it has a negative slope. The full curve is above zero so (Tr1) is rejected. There is an interval fulfilling condition (Tr2) at a low level, but it is not robust to the bandwidth as it occurs where the data is so thin that the confidence interval widens. After $y = 6.5$ a rather strong divergence starts. Most low-income countries have actually entered the Grand Transition; see Paldam (2016) on the growth in Sub-Saharan Africa since the mid-1990s.

At the end of the Grand Transition countries become relatively stable, as will be showed in section 4.4, but the process of transition causes great changes that often include rather unstable periods. The changes often include shifts in the income and power distribution among the major groups of the society, thus, old political alliances are likely to break down and new ones will form.

4.3 The skewness of the Grand Transition in the OPEC group

The horizontal axes on Figures 4 and 5 are the same, but the kernel on Figure 5 is estimated on much fewer observations, so the confidence interval is much wider, and hence the vertical axis is wider. For most of the range – at least for $y > 8.3$ – the two paths are significantly different.

Figure 5. Kernel of the growth-income relation for the OPEC group



Note: See note to Figure 4.

The kernel-curve does not reject that the average curve is a straight line, and it certainly has a negative slope. Also, the negative slope on the OPEC-curve is robust to a wide range of bandwidths (0.1 < bw < 1), and as shown in Table 2 the slope is highly significant. The Democratic Transition has a similar skewness for the OPEC group.

Thus, Figure 5 shows: (i) Very resource rich countries have a much less successful transition than other countries. (ii) They quickly become richer, but then they have less growth. This is precisely as expected from the Dutch Disease story; see, e.g., Paldam (2013) for a survey.

Figure 5 has the intriguing property of *a high-level equilibrium trap*. It shows a stable equilibrium by conditions (Tr1) and (Tr2) for the level of income at $y^* \approx 9.2$. If $y < y^*$, income grows, and if $y > y^*$, income falls. For a large range of bandwidths 0.1 < bw < 0.7 such a point emerges, and it only moves a little. It appears that this observation lacks a theoretical justification, but if it could be made, it may explain, e.g., the amazingly constant income of Venezuela – at approximately that income level – for the last 60 years (even disregarding the present crisis).

4.4 A kernel for the standard deviations

To analyze the variation in the growth rate, we have calculated a *moving standard deviation* $s_k(g)_i$, which gives a $(y_{i(-1)}, s_k(g)_i)$-pair. The *s*-series is made and analyzed as follows:

(1) First the $(y_{i(-1)}, g_i)$-series is sorted by $y_{i(-1)}$. The sorted order is $j = 1$ to N.

(2) Chose an odd k, giving $\kappa = (k\text{-}1)/2$. A sequence is defined as $S_i = (g_{j-\kappa}, ..., g_i, ..., g_{j+\kappa})$, where g_i is the center. The N observations gives $N - 2\kappa$ such sequences.

(3) The sorted g_i-series is supplemented by the standard deviation, $s(S_{ki})$-series. The first κ and the last κ observations in the series are left bank, so that the s-series has Nk = 8,864 − $2\kappa = k - 1$ elements.[9]

(4) Finally, the $(y_{i(-)}, s_{ki})$-series is analyzed by the same kernel-technique as before. The resulting kernel is termed a std_k-kernel. We have calculated the kernel for k = 11, 21 51 – they are strikingly similar. Thus, we only report the std_{21}-kernel.

Figure 6. The std_{21}-kernel on the same data as Figure 3

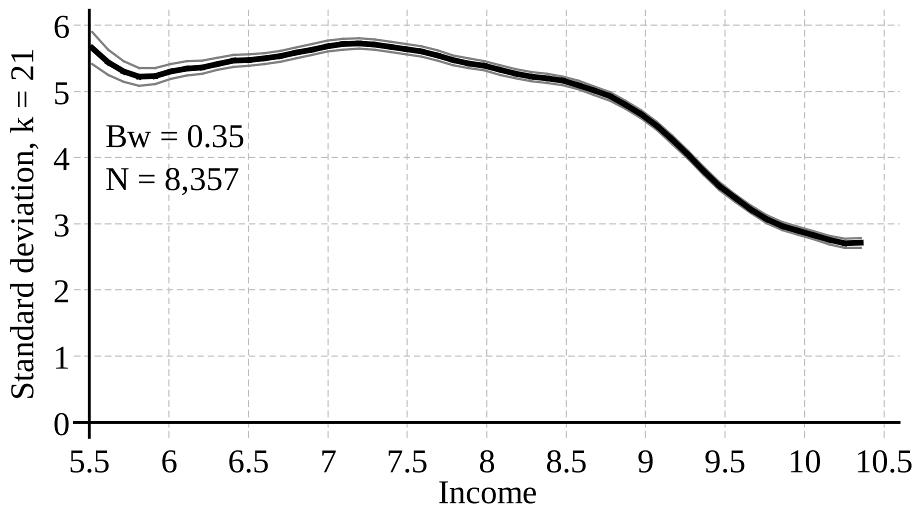


Figure 6 shows that the std_{21}-kernel rises a little from the start till the peak in $y \approx 7.25$. As y increases s_{21} falls. In the end it has fallen to less than half of its peak value. This explains why the confidence intervals on Figure 3 do not fall at the high end, even when the observation density falls. Note also that the confidence interval falls and remains low throughout. The average of the 8,357 s_{21} calculated is 5.06, as appears rather consistent with Figure 3.

9. The s_{11i}-series uses 11 observations for the calculations of the standard deviation. We have also made the calculations with 51 observations. It had no visible effects on the kernel shown. So the std_n-kernel is robust to choice of n as long as N is much larger than n.

5. Simulating the form from Figure 3

The standard way to get a hump-shaped form of the *G*-curve is to return to the two-sector model of development. It is discussed in section 5.1.

In the 1960s and 70s development theory used the observation that in a world with a traditional and a modern steady state, development often took the form of a co-existence of a modern and a traditional sector, which was often seen as an international industrial sector and a traditional agricultural sector. Development then consisted of the gradual replacement of the traditional sector by the modern one. This insight was due to Lewis (1954) and Ranis and Fei (1961). The two-sector model of development was developed in many ways, but then it largely disappeared. However, it has recently reappeared in a version that takes a number of the recent results from the theory of growth in consideration, see Lucas (2009).[10]

5.1 The two sector model of development

To explain the hump-shaped growth pattern, a two-sector model is used where a large agricultural sector in combination with international and domestic spillover effects (externalities) delays the instant growth miracle that would otherwise emerge from the advantage of backwardness. We use the same model for our simulations but calibrate it differently in order to generate the hump-shaped growth pattern reported in Section ???, which peaks at a rate of about 2.5% over an income range of int-$ 3000 - 8000.

In the Lucas (2009) model, all countries are represented as one-factor "AK" economies (Rebelo 1991), where GDP per person is proportional to the single factor input. The labeling of the single factor input is not essential. As used below, K is labeled *capital* and may be interpreted as a broad concept of knowledge capital that also includes human capital.

Output and factor input of the leading economy are denoted in capital letters, such that

(1) $$Y = BK \,,$$

where Y is GDP (income) per person, K is capital per person, and B is a constant. In the leading economy, the stock of capital and per capita income are assumed to grow at the exogenous steady state growth rate μ, which is given by

10. This section closely follows Lucas (2009, section III); see also Gundlach and Svendsen (2016).

(2) $K(t) = K(0)e^{\mu t} \quad \Rightarrow \quad \mu = \dot{K} / K = g_Y$.

In any other (follower) economy, the stock of capital is assumed to evolve according to

(3) $dk / dt = \dot{k} = \mu\, k^{1-\psi}\, K^{\psi}$,

with ψ as an externality linked to the accumulation of capital.[11] Equation (3) implies that the growth equation of a follower economy is given by

(4) $g_y = \dot{k} / k = \mu \left(K / k\right)^{\psi}$.

Hence the growth rate of any follower economy positively depends on the proportional income distance to the leading economy $\left(K / k\right)$ and on the unknown size of the capital gap externality ψ . For any positive value of ψ , equation (4) implies a log-linear relation between the growth rate and the initial capital (respectively income) of a follower country: a larger capital gap relative to the leader implies a higher growth rate of the follower, thereby implying a process of unconditional catching up that is not supported the data discussed in Section ??. Moreover, the Kernel regression results point to a hump-shaped rather than to a linear relation between the growth rate and the initial level of income, which cannot be explained by the one-sector work-horse model of growth empirics.

Which factors may be responsible for a hump-shaped growth pattern? Lucas (2009) argues that a large agricultural sector may act as a brake on catch-up growth because poor, largely illiterate, feudal societies at early stages of development may not be able to implement technology that is available from more advanced countries, at least not as long as a critical amount of capital has not been accumulated. Lucas (2009) formalizes this hypothesis with a model that has "city" (c) and "farm" (f) as two sectors, which both produce a single output good that adds up to GDP.

Since cities are held to be the centers of intellectual exchange, they may generate positive externalities in the sense that the growth rate of a follower economy is affected by an agglomeration externality and by a sectoral productivity externality. City output y_c is produced with capital according to

[11]. In Lucas (2009), ψ is interpreted as an index of openness to trade and technology inflows, where $\psi = 0$ represents a completely closed economy and $\psi = 1$ represents a completely open economy. In Gundlach and Svendsen (2016), ψ is interpreted as an index of social trust.

(5) $$y_c = k(1-x) ,$$

where x is the fraction of the labor force that is employed in agriculture, with $x < 1$. The *agglomeration externality* ζ enters the growth equation (4) as

(6) $$g_y = \mu(1-x)^{\zeta} (K/k)^{\psi} ,$$

with $\zeta > 0$ as a parameter of unknown size.

The *sectoral productivity externality* ξ enters the growth equation (6) via the production function of the farm sector, which is given by

(7) $$y_f = Ak^{\xi}x^{\alpha} ,$$

where y_f is farm output, A is a constant that includes farm land per person, α is labor's share in farm production, and $\xi > 0$ is a parameter of unknown size. Solving equation (7) for the employment share of the farm sector,

(8) $$x = \left(\alpha A / k^{1-\xi}\right)^{1/(1-\alpha)}$$

and inserting equation (8) into equation (6) gives the augmented growth equation of a follower economy as

(9) $$g_y = \mu\left[1-\left(\alpha A / k^{1-\xi}\right)^{1/(1-\alpha)}\right]^{\zeta} (K/k)^{\psi} .$$

The growth path described by equation (9) exhibits a number of distinct features. For instance, a positive growth rate can only result if the term in brackets (the added agglomeration externality) is positive, which is guaranteed by assuming a minimum amount of k that is large enough to allow for any growth at all. If k remains below the critical level, the predicted growth rate approaches 0 with $k \to 0$, all else constant. But if k is increasing beyond the critical level, the growth rate will reach a maximum where the growth drag of a large farm sector is overcompensated by the capital gap. With $k \to \infty$, the term in brackets and the term in parentheses will both approach 1 due to a fading capital gap, which implies that the growth rate of the follower economy (g_y) will gradually approach the growth rate of the leading economy (μ).

Hence equation (9) predicts a hump-shaped pattern of long-run growth: a low growth rate at low levels of capital, a rising growth rate after a critical level of capital has been reached, and a gradually declining growth rate over the subsequent process of catching up with the leading economy in the long-run. The growth advantage of extreme poverty implied by the textbook Solow model and the one-sector model of equation (4) is eliminated by the introduction of a second sector that generates further externalities. What remains to be seen is whether reasonable parameterizations of equation (9) can actually generate a hump-shaped growth pattern for a hypothetical economy that resembles hump-shaped growth pattern identified by the Kernel regressions of Section ??.

5.2 Simulating hump-shaped growth

Simulating the growth path of an initially poor hypothetical economy on the basis of equation (9) requires initial values for per capita income in the leading (Y_0) and the following economy (y_0) and for the employment share of agriculture (x_0) as well as parameter values for μ, ψ, ξ, ζ, α, and A.

The initial level of per capita income of the leading economy is set to $Y_0 = 12,000$ (int. \$), which equals about the income level of the United States in 1960. The initial level of per capita income of the following economy is set to $y_0 = 1,000$ (int. \$), which is close to the average income level of poor though not extremely poor economies in 1960. Hence the initial income gap between the leading and the following economies is set to 8.3%. The constant growth rate of the leading economy is set to 1.8%, which reflects the average annual growth rate of per capita income of the United States in 1913-2010 (Maddison Project data: Bolt and van Zanden (2014)).

The other parameter values are more difficult to motivate. Lucas (2009) employs equation (8) to describe the correlation between the employment share of agriculture (x) and log GDP per capita (y) in cross-country and time series data and finds a best fit by setting $\xi = 0.75$, $\alpha = 0.6$. For given ξ and α, the implied parameterization of A can be derived for a given value of k_0, which itself can be derived from the equations of the model for an initial income level y_0. With $y_0 = 1,000(\text{int}.\$)$ as noted above, we derive parameterizations of $h_0 = 714(\text{int}.\$)$ and $A = 5.06$.[12]

The parameterization of the capital gap externality, ψ, is based on a theoretical range [0,1], where a value of 0 would apply for an economy that is completely closed to any spillovers from leading economy, say, in the form of imports of advanced capital goods and technology, while a value of 1 would apply for an economy that can fully capitalize on the gains from backwardness, as assumed by the textbook Solow model. Real economies do not fall into these theoretical border cases, so we limit the range of parameterizations to [0.2, 0.4, 0.6], with $\psi = 0.4$ as our preferred parameterization for the average sample economy in 1960.

These settings leave the agglomeration externality (ζ) as the only unknown parameter.[13] Since empirical evidence on the size of ζ is missing, we employ the alternative parameterizations [0.6, 1.2, 1.8][14] in order to match the hypothetical growth path with the empirical hump-shaped growth path reported in Figure ?? [Kernel].

Figure ? shows simulated growth paths of a hypothetical poor economy with an initial employment share of agriculture of $x_0 = 60\%$ and a capital gap externality of 0.4.[15] The different paths represent the alternative parameterizations of ζ for a given set of other parameters. A hump-shaped growth pattern emerges for all three cases considered, but it looks that parameterizations with $\zeta < 1.2$ tend to generate a growth path that is close to the log-linear path predicted by the one-sector growth model (the textbook Solow model) while parameterizations with $\zeta > 1.2$ tend to generate delayed growth miracles that are also not supported by the empirical record. For $\zeta > 1.2$, the simulated growth path looks similar to the observed growth path. With an initial income level of \$ 1,000, the simulated growth rate starts below 2% but rises steadily until it peaks close to a rate of 3% after about 85 years, when the accumulated income level of the hypothetical economy has reached about int-\$ 8000. Subsequently, the growth rate gradually falls until it approaches the long-run steady state growth rate of 1.8%.

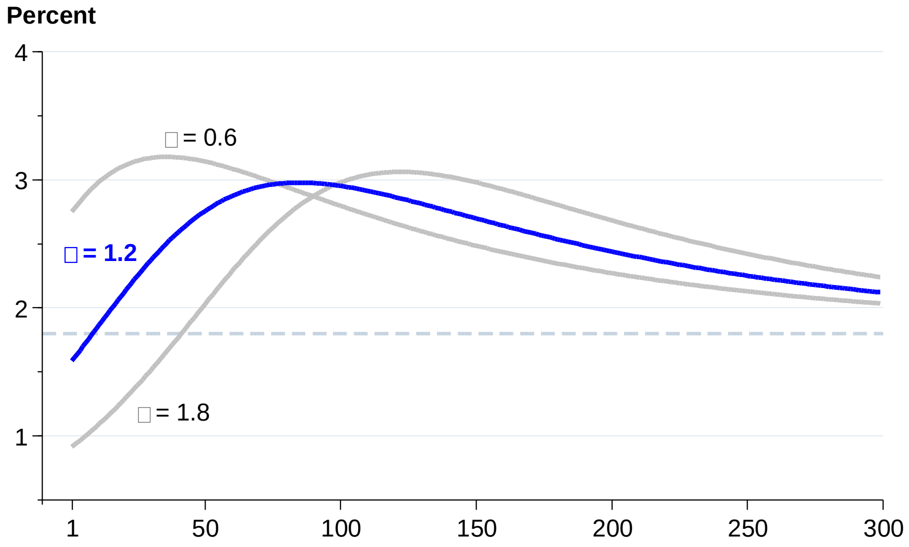
12. Detailed results are available upon request; see also the calculations in the appendix of Gundlach and Svendsen (2016).

13. For $\zeta = 0$, equation (9) equals equation (4).

14. Lucas (2009) uses $\zeta = 1$ as his preferred parameterization.

15. The simulation results are generated with Stata. The code is available upon request.

Figure 7. The simulated growth paths for different values of the agglomeration externality



Notes: Growth in % of hypothetical economy over time. Parameters ψ = 0.4, ς as indicated, α = 0.6. Initial conditions: y(0) = 1,000 and x(0) = 0.6.

Independent from any specific parameterization, the underlying model always generates a growth miracle and income convergence in the very long run. But alternative parameterizations determine the timing and the size of the simulated growth miracles, which can be compared to observed growth miracles. Figure 7 provides a robustness check in the form of a variation of the capital gap externality (ψ), holding constant the value of the agglomeration externality at $\zeta = 1.2$.

6. Conclusions

The above analysis is done using a technique chosen to assume as little theory as possible. We hope the reader will agree that the analysis builds on very few assumptions. This will allow us to draw at least one rather strong conclusion about economic theory: It does appear that there is a transition in the growth rate: From moderate to higher and back again to moderate. Thus the one-sector steady state perspective on economic growth is problematic.

In addition our findings tell a story about the long-run dynamics of income of the world system of countries:

Poor countries (LICs) have a low and unstable growth. However, the growth is still at an average rate of 1.6% per year. The stagnating traditional economy is all but gone today.[16]

Rich countries (HICs) have a growth rate of 1.6% p.a. as well. The instability of this growth is half the one of the LICs.

Middle-income countries (MICs) have, in average, 1 percentage point higher growth than in the LICs and HICs. The peak is rather flat between y = 8 and 9. During that period the variability of the growth rates falls.

An excess growth of 1 percentage point – as is found for the MICs – accumulates to one logarithmic point over a century. This means that many MICs will actually catch up with the HICs during the next century. It will take considerably longer for the LICs.

Due to the high variability of the growth of the LICs and MICs some countries will catch up much faster than others.

16. The world has about 10 countries with a *gdp* (GDP per capita) that is lower in 2010 than in 1950.

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