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Seven pairs of kernel regressions: looking for causality

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This note compares pairs of kernel regressions between two variables. The data is a balanced, unified sample with 1,051 observations for the five variables:[2] *EF*, *T*, *SC*, *V* and *y*, listed in Table 1. It is the same sample as used in Paldam and Saardaoui (2026), from now *ibid*. [Figure #] refers to a figure in that paper. The first four of the variables are institutional indices.

Part 1 analyzes the relation of (*SC*, *EF*), (*SC*, *T*) and (*SC*, *V*), in sections 1, 2, and 3.
Part 2 analyzes the transitions in *EF*(*y*), *T*(*y*), *SC*(*y*), and *V*(*y*) in sections 4, 5, 6, and 7. These sections ask if the reverse relation works equally well.

Kernel regressions on unified data is a method for revealing common long run trends in cross-country data sets as explained in Paldam (2021 and 2024). The kernel regression that explains *x* by *y* is written $x = K^x(y, bw)$, where *bw* is the bandwidth. K^x is a smooth function of *bw*, so it is normally easy to find the best *bw*. The program (lpoly in stata) starts by a good estimate.

Kernel regressions require large datasets, so the panels are unified by stacking. One variable is thus one column. The elements in the columns are in the same order, but the order is not important.

Table 1. The five variables and their sources

Variable	Source
EF, economic freedom	From the Fraser Institute https://www.fraserinstitute.org
SC, state capture	From Natural Resource Governance Institute https://governanceactionhub.or
T, corruption	From the TI index of Transparency international, T = 10 - TI https://www.transparency.org/en/cpi/2024
V, democracy	The polyarchy index from the V-Dem project https://v-dem.net/
y, income (ln gdp)	Ln gdp that is real GDP per capita. from the Maddison Project https://www.ggdc.net/maddison/maddison-project/home.htm

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[2] OPEC countries are deleted from the sample as the have a different pattern analyzed elsewhere, *ibid.*

However, each kernel regression orders the two columns analyzed by the explanatory variable. $x = K^x(y, bw)$ orders the (x, y) vector by y, while $y = K^y(x, bw)$ orders the vector by x. The two kernel curves differ due to the sorting.

Kernel pairs may provide causal evidence. If one of the two kernel curves looks as predicted by a theory, it is evidence for that theory, and hence for the causality it implies. It is strong evidence if the reverse kernel does not look like anything predicted by a theory. It is also possible that both curves in the pair look equally good – this suggests simultaneity.

When the correlation between the variables is high there is always some reflection of the best of the two kernel curves on the other. Thus, we look for the main connection and the weak reflection.

Tables 2 and 3 give some descriptive statistics used below. The two sets of correlations – Pearson and Spearman – are fairly similar. Thus, the distributions of the variables will be disregarded from now. The four institutional indices are compiled in two different ways. *SC* and *T* are relative due to annual calibrations, having no international trends. *EF*, *V*, and *y* are absolute measures with international trends. This should influence the correlations, but it is not easy to compensate the correlations for that difference in construction.

Kernels provide no R^2-scores, but a graph. We measure how much they explain the range of the data from Table 2, compared with the range of the curve as shown by the vertical arrow at the right-hand side of the graph. The relation of the two is reported as "ratio".

Table 2. Descriptive statistics for the five variables. All have $N = 1{,}051$

Variable	Mean	Std. dev.	Min	Max	Range
EF	6.74	1.11	3.04	9.23	6.19
SC	44.22	24.77	2.43	94.32	91.88
T	5.49	2.14	0.10	8.80	8.70
V	0.59	0.25	0.08	0.92	0.85
y	9.15	1.20	6.08	11.37	5.29

Table 3. Correlation of the unified data – representing the long run

	r, Pearson correlations					ρ, Spearman correlations					
	EF	*SC*	*T*	*V*	*y*		*EF*	*SC*	*T*	*V*	*y*
EF	1					*EF*	1				
SC	-0.70	1				*SC*	-0.72	1			
T	-0.77	0.84	1			*T*	-0.80	0.87	1		
V	0.66	-0.84	-0.67	1		*V*	0.70	-0.86	-0.73	1	
y	0.76	-0.68	-0.77	0.62	1	*y*	0.81	-0.72	-0.82	0.67	1

The relevant r correlations are quoted in the headlines to all sections below.

Part I. Three relations between the SC-index and other institutions

1. (*SC*, *EF*) pair of state capture and economic freedom, r = −0.70

Figure 1a. *EF*(*SC*), economic freedom explained by state capture. Ratio: 2.8/6.2 = 0.45

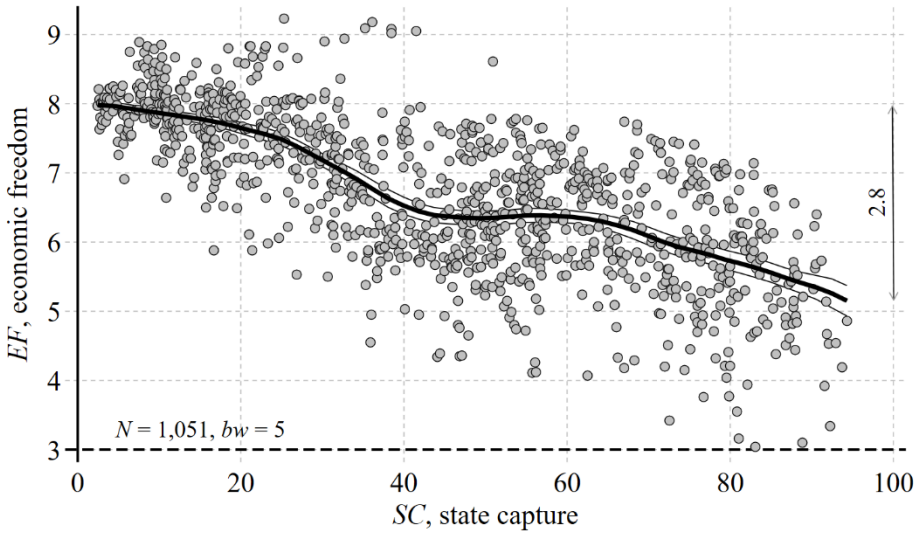
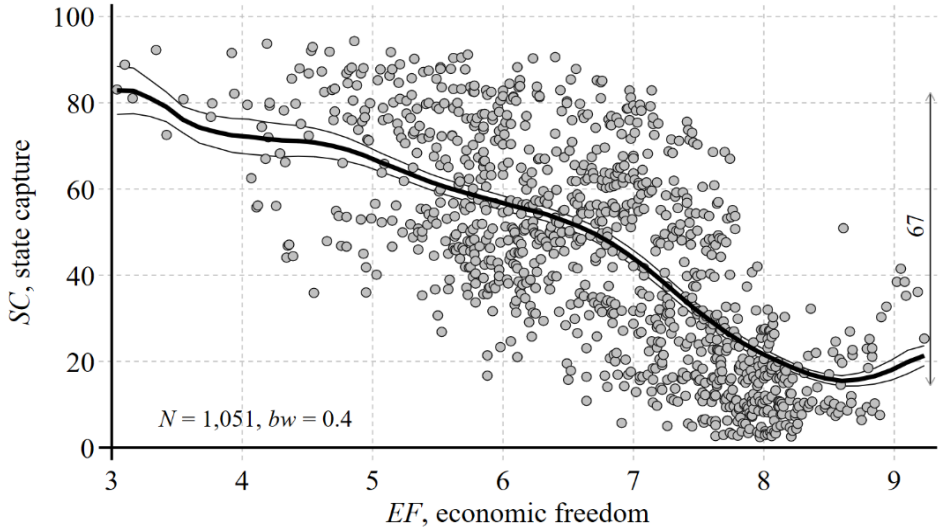


Figure 1b. *SC*(*EF*), state capture explained by economic freedom. Ratio: 67/92 = 0.73



It is argued that market capitalism and political capitalism is alternatives so if the *EF* and *SC*-measures were perfect they should have a correlation of -1, *ibid*. Thus, the two graphs are difficult to interpret.

Figure 1a shows that *SC* explains 45% of the range by a fairly linear curve. However economic freedom explains 73% of the range of *SC*, by an interesting curve with two bends. This argues that economic freedom better explains political capitalism than vice versa, but as mentioned it may also, in some way, reflect the quality if the indices.

2. (*SC*, *T*) pair of state capture and corruption, r = 0.84

Figure 2a. *T*(*SC*), corruption explained by state capture. Ratio: 6.3/8.7 = 0.72

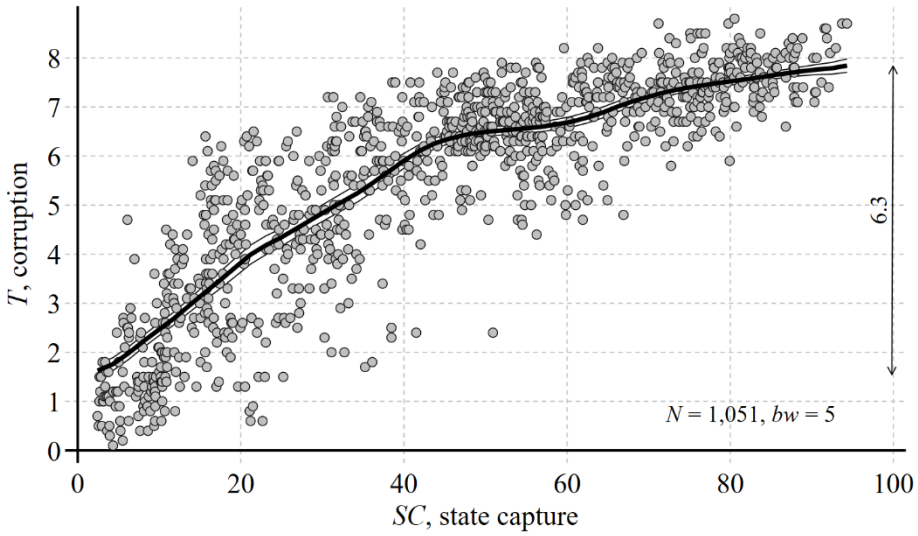
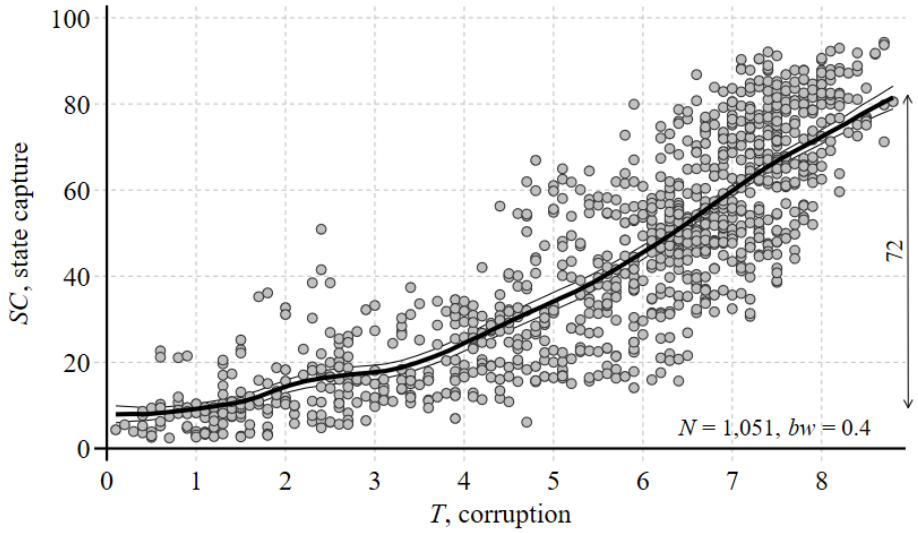


Figure 2b. *SC*(*T*), state capture explained by corruption. Ratio 72/92 = 0.78 [Figure 7]



Corruption is $T = 10 - TI$, where *TI* is Transparency Internationals corruption index, which measures honesty.

Here the two curves look very similar though, of course, they bend the opposite way. Also, they explain almost the same fraction of the range. This indicates simultaneity. Corruption is a typical part of the triplets of political capitalism, state capture, and crony capitalism, see *ibid*. Thus, we are looking at the relation between the whole and a part.

3. (*SC*, *V*) pair of state capture and democracy, r = −0.84

Figure 3a. *V*(*SC*), democracy explained by state capture.
Ratio: 0.66/0.85 = 0.78

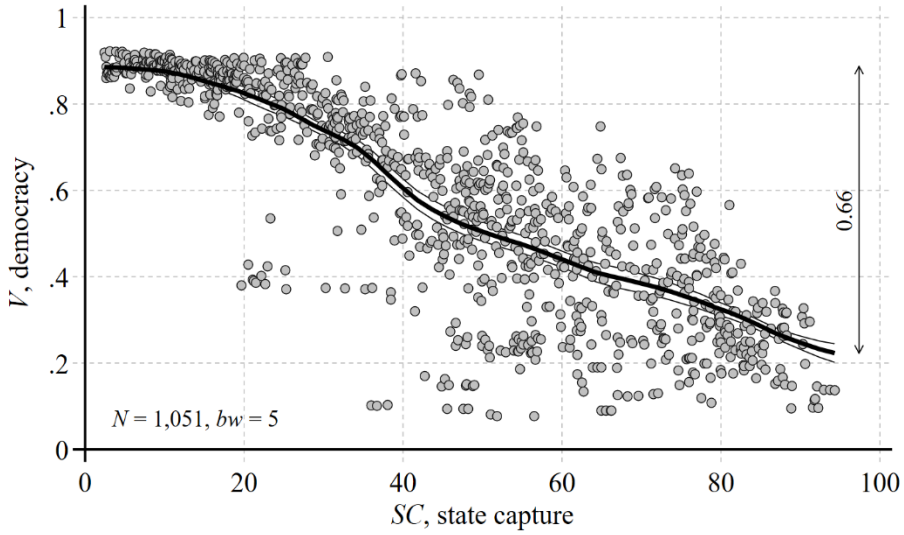
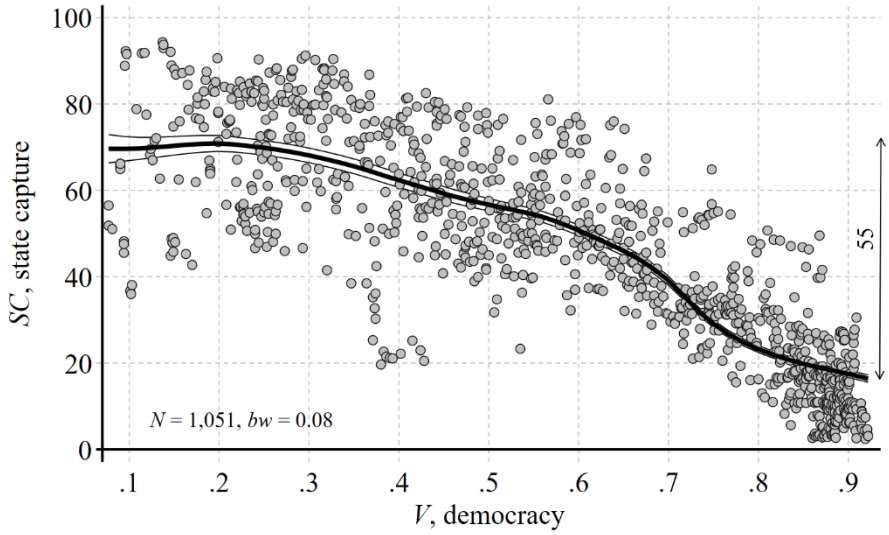


Figure 3b. *SC*(*V*), state capture explained by democracy.
Ratio: 55/92 = 0.60
[Figure 8]



Democracy is the polyarchy index from the V-Dem project. The two indices have a correlation of -0.84, of which about half is a within country correlation, which takes place in less than 25 years.

Here the story is much the same as in section 2, so once again it points to simultaneity, but there is a weak indication that the main direction of causality is from state capture to democracy.

Part II. Four transitions with control for reverse causality

Transition theory predicts that the kernel curve looks as ⌒ when r > 0 and ⌣ when r < 0.

4. EF transition: (*EF*, *y*) pair of economic freedom and income, r = 0.76

Figure 4a. *EF*(*y*) transition, Economic freedom explained by income. Ratio: 3.3/6.2 = 0.53 [Figure 5]

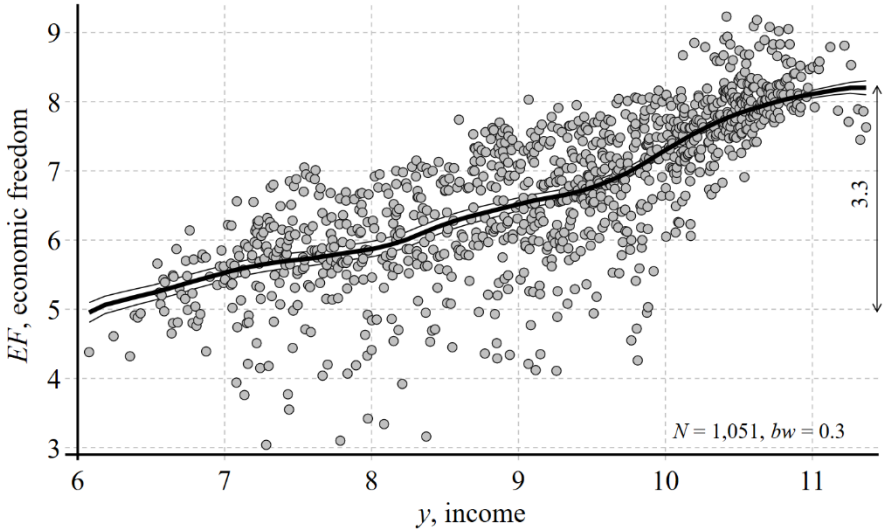
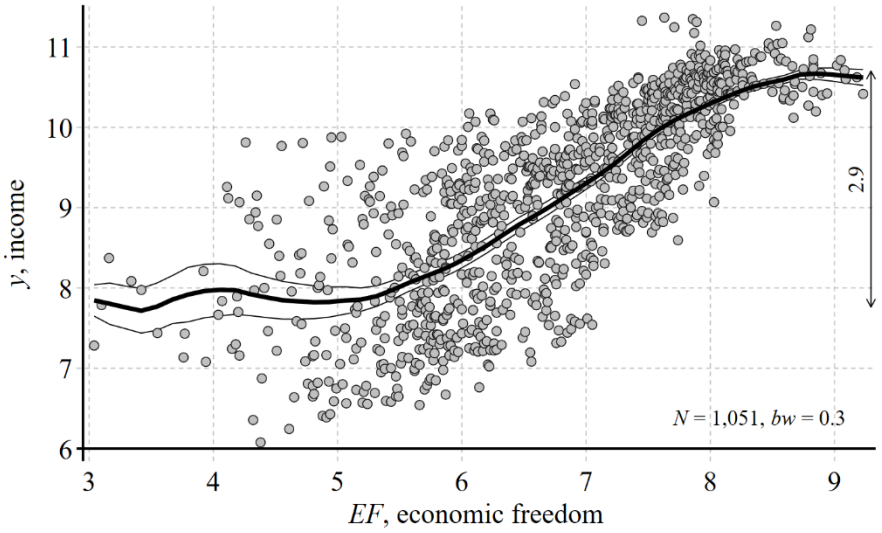


Figure 4b. *y*(*EF*) reverse, income explained by economic freedom. Ratio: 2.9/5.3 = 0.55



The curve on Figure 4a has a more linear shape than predicted by transition theory, but it certainly shows that income increases the *EF* variable. Figure 4b supports the claim of the Fraser Institute that economic freedom makes societies wealthy. The claim is shown to work only for *EF* > 5.5.

Figures 4a and 4b explain the same fraction of the range. So, it is not clear which interpretation to prefer.

5. *SC* transition: (*SC*, *y*) pair of state capture and income, r = −0.68

Figure 5a. *SC*(*y*) transition, state capture explained by income.
Ratio: 56/92 = 0.64
[Figure 5]

Figure 5b. *y*(*SC*) reverse, income explained by state capture.
Ratio: 2.6/5.3 = 0.49

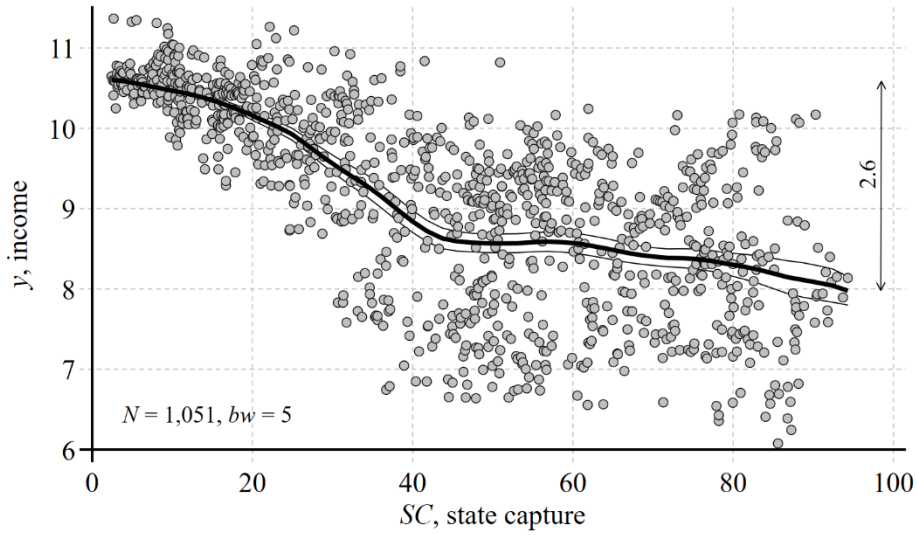


Figure 5a looks as it should by transition theory, and the ratio is larger on Figure 5a than on Figure 5b. This supports transition theory. Figure 5b is interpreted as the weak reflection of the transition curve.

6. *T* transition: (*T*, *y*) pair of corruption and income, r = −0.77

Figure 6a. *T*(*y*) transition, corruption explained by income.
Ratio: 5.5/8.7 = 0.63
[Figure 4 and 5]

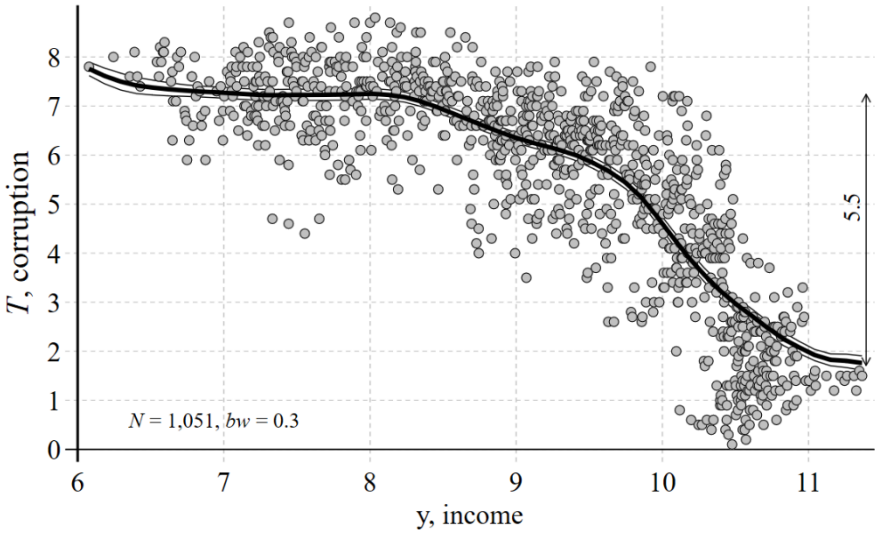


Figure 6b. *y*(*T*) reverse income explained by corruption.
Ratio: 3/5.3 = 0.57

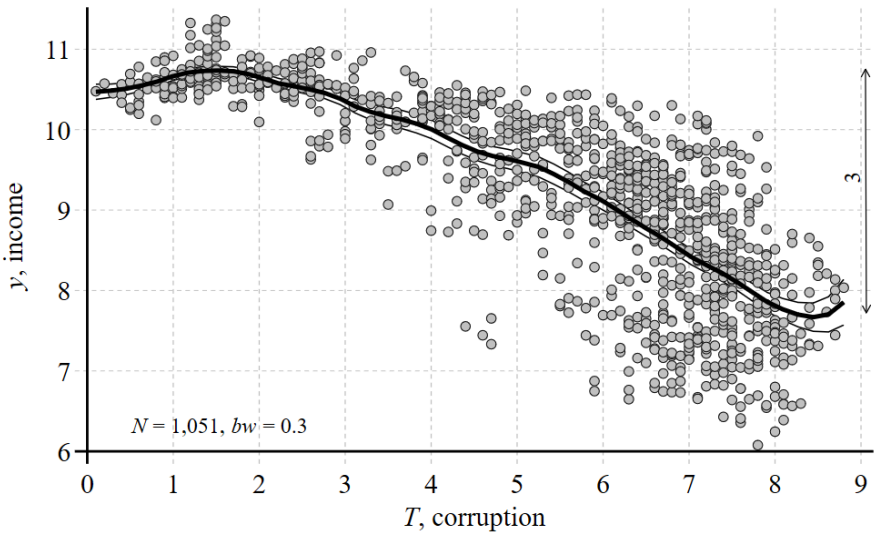


Figure 6a looks as it should by transition theory, and the ratio is a bit larger on Figure 6a than on Figure 6b. This supports transition theory. Figure 6b is interpreted as the weak reflection of the transition curve

7. *V* transition: (*V*, *y*) pair of democracy and income, r = 0.62

Figure 7a. *V*(*y*) transition, democracy explained by income. Ratio: 0.43/0.85 = 0.51 [Figure 5]

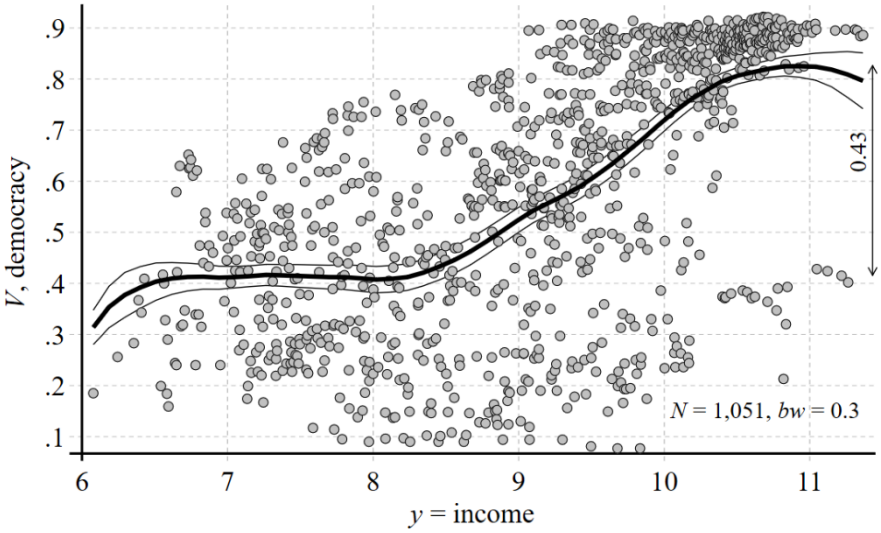


Figure 7b. *y*(*V*) reverse, income explained by democracy. Ratio: 1.8/5.3 = 0.34

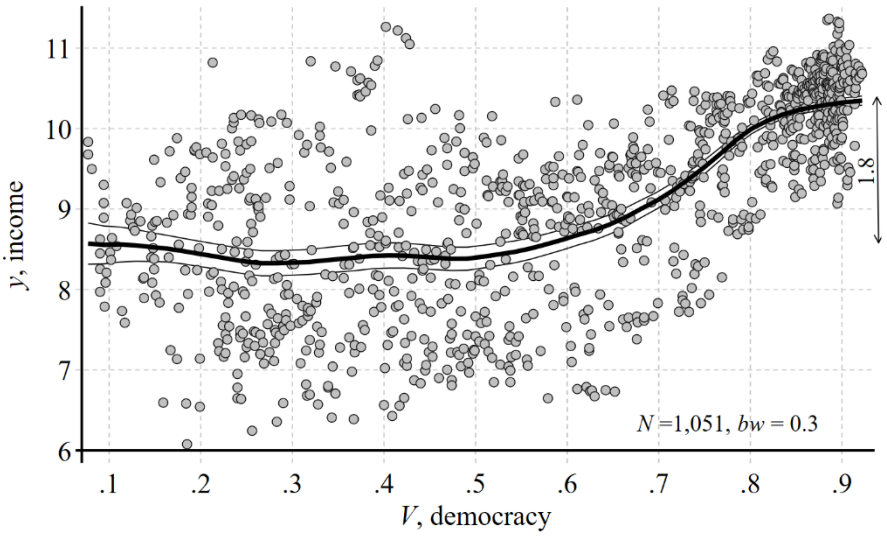


Figure 7a looks as it should by transition theory, and the ratio is larger on Figure 7a than on Figure 7b. This supports transition theory. Other work shows that the transition curve is very robust to the sample. Figure 7b is interpreted as the weak reflection of the transition curve.

Figure 7b explains little of the range. While the transition curve is flat from *y* = 6 to 8,5, which is less than half of the *y*-range, the reverse curve is flat from *V* = 0.05 to 6 which is 2/3 of the range. Income explains democracy much better than vice versa.

8. Conclusion

The variables are four institutional indices and income. The four indices are compiled by four independent NGOs with headquarters in different countries. Institutional indices have substantial measurement uncertainties such as 10%. Nevertheless, they have (numerical) correlations that in average is about 0.75. Most of the correlation is explained by the cross-country variation.

Three of the four institutional series (*SC*, *T*, *V*) have strong and very similar transitions. The last institutional index (*EF*) has a weaker transition. However, [Figure 5] (ibid) show that all four transitions are very similar. This gives the institutional series a strong underlying confluence. It argues that the main causal direction is from income to the institutions.

The analysis shows that state capture and corruption have a strong and simultaneous connection. The connection is due to the cross-country pattern. State capture and democracy are also mainly simultaneous, though there are signs that the causality from state capture is strongest. Here, the short run (within 25 years) connection is half on the correlation.

Finally, state capture and economic freedom are two alternative types of capitalism that should add to 100% so the correlation should be -1 per definition. The reason it is only -0.7 is measurement uncertainty as discussed in

References: (MP is Martin Paldam, JS is Jamel Saadaoui

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