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Version 2

# Church density and the church membership ratio

## The cross-municipal pattern in Denmark, 2012

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Abstract: The church density is the number of churches per inhabitant. The membership ratio is church membership in percent of the population. The paper is a statistical analysis of the Danish cross-municipal pattern in the two variables for 2012. Nine explanatory variables are used: Area, population, income (tax base per capita) and six regions. Four of these explanations work: Area, population, income and the capital region. Income and capital region have strong multicollinearity. The four variables explain both church density and the membership ratio. Once they are included, there is no interaction between membership and church density.

Content



Note: This paper is a background paper in a project listed in the references. The paper is written in English to allow cross references to the other papers in the project although the subject matter is mainly of interest to Danes. All data used are reported in the Appendix.

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# 1. Introduction

This paper is a statistical analysis of the Danish cross-municipal pattern in 2013 of the church density (*Lndens*) and church membership (*Mprc*), where I have also added a time series perspective. The church is the Lutheran Church that is the 'official' Danish Church

### *1.1 Two outcomes and nine explanations*

The variables analyzed are defined in Table 1. The two outcome variables *Lndens* and *Mprc* are assumed to be functions of three quantitative variables, area, population and income, and a set of six qualitative (binary) variables for the regions of the country. It is divided into five administrative regions and I have added one extra 'region' by singling out five islands.

Table 1. The variables for the 98 municipalities

| Variable | Definition and scaling. The data are reported in Appendix | |
|---|---|---|
| | **Outcome: Church density (per 10,000 inh.)** | |
| *Lndens* | Natural logarithm to *Dens* | |
| *Dens* * | Density, 10,000 times *Net* divided by Population | |
| *All* * | All churches in municipality, include churches in institutions | |
| *Net* * | Parish churches, *All* net of institutional churches | |
| | **Outcome: church membership ratio (of population)** | |
| *Mprc* * | *Mem* as share of *Pop* (both scaled the same) | |
| *Mem* | Membership in the Danish People's Church | |
| | **Quantitative explanations (natural logarithms, ln)** | |
| *Lnarea* | Ln *Area* (before scaling) | |
| *Lnpop* | Ln population (*Pop* before scaling) | |
| *Lntb* | Income, ln tax base (*Tb*) | |
| *Area* * | Area in $km^2$ scaled by division by 100 | |
| *Pop* * | Population 2012 scaled by division by 10,000 | |
| *Tb* * | Tax base per capita | |
| | **Qualitative explanations: Administrative division** | Number |
| *Cap* * | Capital Region, Bornholm is *Island* | 28 |
| *Island* * | Five islands that belong to regions, but may differ | 5 |
| *MidJ* * | Central Jutland Region, Samsø is *Island* | 18 |
| *NorJ* * | North Jutland Region, Læsø is *Island* | 10 |
| *South* * | South Denmark Region, Fanø and Ærø are *Islands* | 20 |
| *Zea* * | Zealand Region | 17 |

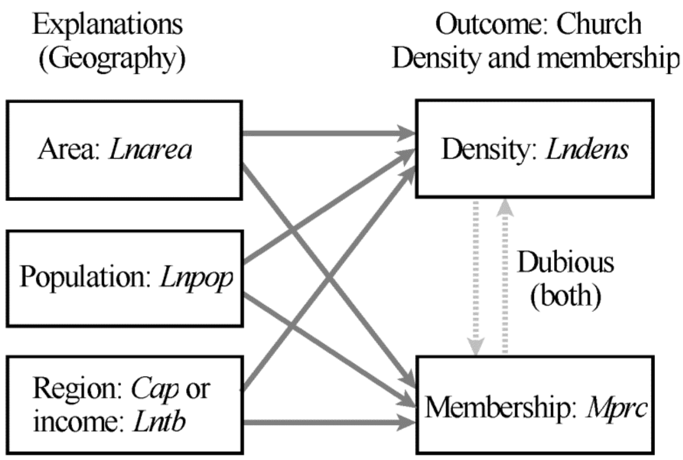
Note: The rows shaded in gray are used to calculate the variables analyzed. *Island* is not an administrative division but consists of the islands listed. The numbers of municipalities reported are net of islands, so all numbers add to 98, as they should. The qualitative variables are binary dummies, which are 1 for municipalities in the region and 0 otherwise. The * indicates data reported in Table A1 of the Appendix.

The quantitative variables have rather skew distributions as shown in section 2, so three of the variables are given a logarithmic transformation. This improves the fit by re-weighting extreme observations, but the distributions are still non-normal.

### *1.2 The model*

The analysis assumes that causality is straightforward: The geographic variables may cause the outcome variables, but not vice versa.

Figure 1. The main causal structure found




The paper analyzes the causal structure illustrated on Figure 1:

(i) The causal links, depicted by dark gray arrows, from the geographical variables to *Lndens* and *Mprc*. Only one of the regional dummies works throughout: It is *Cap* for the capital region. Thus, three geographic variables, *Lnarea*, *Lnpop* and *Cap*, cause church density, and they are also the main variables explaining church membership. *Income* is so related to *Cap* that the two variables can replace each other in explaining the outcomes, though the story is a bit more complex for *Mprc* than for *Ldens*.

(ii) The causal links, depicted as the dotted light gray arrow on the figure, from *Mprc* to *Lndens* and vice versa. Even when the two outcomes are highly correlated this turns out to be because they have the same explanations – links (i) dominate links (ii).

### *1.3 One part of a larger project on the religious transition*

The statistical analysis is strictly cross-municipal, and thus it is a macro analysis. Obviously, one may want to dig deeper and explain the micro theory behind the macro relations, but this is largely outside the frames of the analysis. However, the paper is a minor part of a project with two major parts (see references) analyzing the *religious transition*: It is the large fall in religiosity caused by the modern economic development that started two centuries ago.

The religious transition is one part of the grand transition, which is the large change in society that has taken place in Western Europe and which is gradually spreading to the rest of the world. Apart from the religious transition it has caused: (a) a twenty-fold increase in production per capita, (b) a five-fold increase in population, (c) a strong urbanization, etc.

Part one of the project analyzes the cross-county pattern in religiosity. This part of the project is published. Here the transition appears when poor and rich countries are compared. The analysis is done using 14 polled religiosity indicators for 95 countries covered by the World Values Survey. One of these is church membership. While church membership is negatively correlated to income, it is the variable with the (numerically) lowest correlation.

Part two of the project concentrates on the *Dens*-variable by using church data for Denmark starting well before modern economic development. The data are not yet complete, but it is clear that they will show a large fall in *Dens* that is explained by the religious transition. It also shows that few churches have been closed since 1800.

In terms of the grand transition incomes have grown more in the cities – notably in the capital region – and this has caused a large-scale urbanization leaving the rural areas with a falling or stagnating population.

These processes explain the skew distribution of churches: They can either be explained by income or by movements in the population: Churches in areas with a falling or stagnant population have not been closed, but few new ones have been needed in the areas where the population has grown, notably in the new suburbs to Copenhagen.

The *Mprc*-data show a much smaller variation as it appears that the religious transition has been much slower in this variable. See section 6 for a discussion of this variable, with a look backward and forward on the short time series.

## 2. Variables and correlations

The variables analyzed were defined in Table 1. The main series are given in the appendix table allowing the reader to check and replicate everything.

### *2.1 Descriptive statistics – Table 2 and Figure 2*

Table 2 contains descriptive statistics which allow a couple of observations:

Firstly, the last row of Table 2 brings p-values (in %) for the skewness/kurtosis test of normality, which is always rejected. They are a little closer to normality after a logarithmic transformation. Regression results are fairly robust to non-normality, but the main results have to be checked for the effects of outliers.

Secondly, it appears that the *Dens*-variable varies 7½ times from 2 to 15, which is much more than the *Mprc* that only varies 1½ times from 0.6 to 0.9.

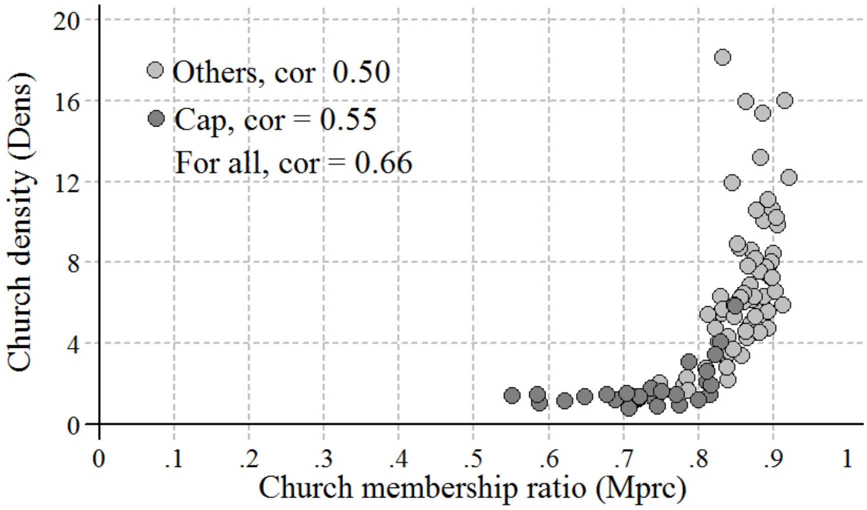
The relation between the two outcome variables is analyzed in Figure 2. The variables are related, but the average curve through the points is neither proportional nor linear. The figure distinguishes between the observations from the *Cap*-region and others by two shades of gray also used in later figures. The observations from the *Cap*-region are on an almost horizontal line, while the other observations are on an almost vertical line. This pattern does not change if the *Lndens* is used instead of *Dens.*

Table 2. The qualitative variables – descriptive statistics

| | The original data in sources (1) – (6) | | | | | | The transformed variables (7) – (11) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| | *All* | *Net* | *Dens* | *Area* | *Pop* | *Tb* | *Lndens* | *Mprc* | *Lnarea* | *Lnpop* | *Lntb* |
| National values | 2.422 | 2,402 | 4.30 | 42,895 | 5,584,688 | 162,938 | 1.46 | 0.798 | 10.67 | 15.536 | 12.001 |
| | Cross-municipal data (averages disregard size of municipal) | | | | | | | | | | |
| Mean | 24.71 | 24.51 | 5.40 | 441.19 | 56,986 | 164,114 | 1.41 | 0.824 | 5.47 | 10.64 | 12.008 |
| Median | 21 | 21 | 4.88 | 361.49 | 43,554 | 154,222 | 1.58 | 0.848 | 5.89 | 10.68 | 11,946 |
| Std Deviation | 18.02 | 17.94 | 3.80 | 375.68 | 65,327 | 27,855 | 0.79 | 0.079 | 1.37 | 0.79 | 0,147 |
| Smallest | 2 | 2 | 0.79 | 8.10 | 1,880 | 137,498 | -0.24 | 0.551 | 2.09 | 7.54 | 11,831 |
| Largest | 86 | 82 | 18.10 | 1488.86 | 551,580 | 288,553 | 2.90 | 0.920 | 7.31 | 13.22 | 12,573 |
| Skewness | 0.87 | 0.82 | 1.05 | 0.78 | 5.30 | 2.43 | -0.31 | -1.46 | -0.82 | -0.71 | 3.94 |
| Kurtosis | 3.57 | 3.35 | 4.01 | 2.94 | 37.26 | 6.87 | 2.01 | 1.87 | 2.57 | 6.96 | 1.89 |
| Normality (%) | 0.43 | 0.82 | 0.06 | 1.62 | 0.00 | 0.00 | 0.23 | 0.00 | 0.96 | 0.01 | 0.00 |

Note: All skewness/kurtosis normality tests are below 5 %, so they reject normality. The area data does not aggregate perfectly as some (minor) areas are under state jurisdiction.

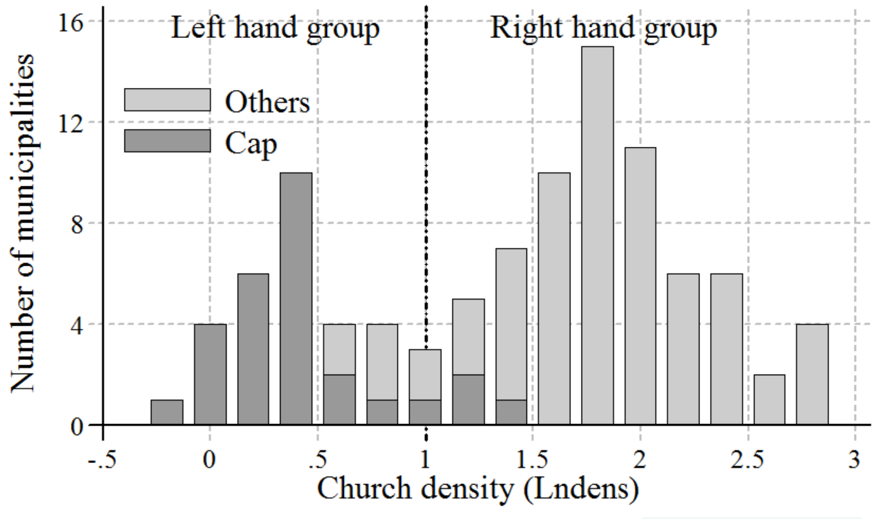
Figure 2. The outcome variables: The scatter of *Dens* over *Mprc*




*2.2 The distribution of the two outcomes and income, Figures 3, 4 and 5*

Figure 3 shows the distribution of *Lndens*. When the same figure is calculated for *Dens* it has a long tail to the right. The distribution of *Lndens* has two peaks: The left hand group for *Lndens* < 1 largely consists of the municipalities in the *Cap* (Capital) Region while the right hand group for *Lndens* > 1 are all other municipalities. Appendix Table A2 lists the exceptions to this picture. Both groups seem to be normally distributed.[2]

Figure 3. The distribution of *Lndens*, the logarithmic church densities




2. Figure 3 suggests that if a number *x* is added to *Lndens* for the municipalities in the Capital region, normality results. This has been done for *x* = 0.6, 0.7, … , 1,6 for three normality tests: Skewness-Kurtosis (1.14), Shapiro-Wilks (1.05) and Shapiro-Francia (1.01), where the number in brackets is the best *x*. In all cases normality is accepted at the 50% level for all *x* in the interval from 0.9 to 1.3.

Figure 4 is drawn as Figure 3, but for *Mprc*. It shows a less clear picture where the *Cap*-group has a large variation. The picture does not get clearer by taking the logs or the antilog. Figure 5 shows the distribution of the income data that looks more similar to the one for *Mprc* than the one for *Lndens*. However, Figures 6 and 7 show a more complex pattern.

Figure 4. The distribution of *Mprc*, the church membership percentages

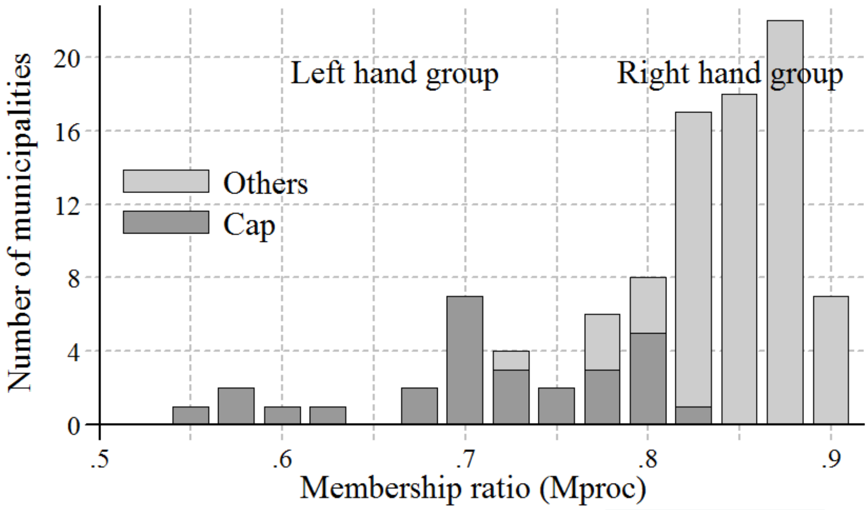
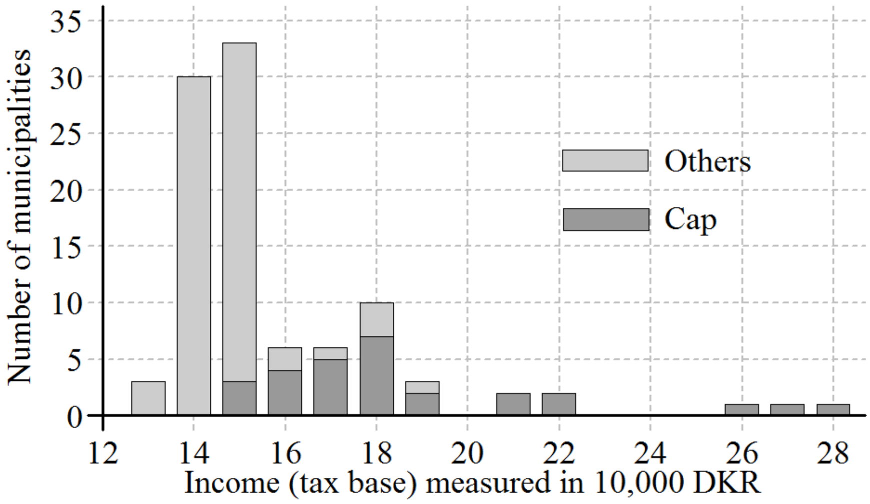



Figure 5. The distribution of *Tb*, the tax base per capita




When Figures 3, 4 and 5 are compared it is not surprising that *Cap* does a good job explaining *Ldens*, while it is a little more difficult to explain *Mprc* – here income also contributes. Section 4 considers the same variables in different representation.

## 2.3 *Looking at the 2 x 10 extreme municipalities*

Table 3 shows the large difference between the municipalities with the 10 lowest and the 10 highest church densities. The 10 lowest are all suburbs to Copenhagen that grew rapidly in the 20$^{th}$ century. The 10 highest are rural municipalities with stagnating population.

Table 3. The extreme densities (*Dens and Lndens*)

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|
| The 10 lowest church densities | | | | The 10 highest church densities | | | |
| Municipality | Region | *Dens* | *Lndens* | Municipality | Region | *Dens* | *Lndens* |
| Hvidovre | *Cap* | 0.79 | -0.240 | Ærø | *Island* | 10.55 | 2.356 |
| Glostrup | *Cap* | 0.92 | -0.080 | Vesthimmerland | *NorJ* | 10.64 | 2.365 |
| Tårnby | *Cap* | 0.97 | -0.029 | Rebild | *NorJ* | 11.08 | 2.405 |
| Albertslund | *Cap* | 1.08 | 0.073 | Lolland | *Zea* | 11.95 | 2.481 |
| Rødovre | *Cap* | 1.08 | 0.080 | Lemvig | *MidJ* | 12.19 | 2.501 |
| Brøndby | *Cap* | 1.17 | 0.160 | Thisted | *NorJ* | 13.15 | 2.576 |
| Gladsaxe | *Cap* | 1.22 | 0.200 | Langeland | *South* | 15.37 | 2.732 |
| Hørsholm | *Cap* | 1.23 | 0.209 | Morsø | *NorJ* | 15.91 | 2.767 |
| Ballerup | *Cap* | 1.25 | 0.221 | Læsø | *Island* | 15.96 | 2.770 |
| Furesø | *Cap* | 1.31 | 0.270 | Samsø | *Island* | 18.10 | 2.896 |

Note: The regions are defined in Table 1. The Appendix reports the region of all municipalities.

The same story applies to the membership ratio: All 10 municipalities with the lowest *Mprc*'s are in the *Cap* region, and all 10 with the lowest *Mprc*'s are more rural municipalities.

This already suggests a story – a story that will unfold as the paper proceeds.

Table 4. The extreme membership ratios (*Mprc*)

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| The 10 lowest membership ratios | | | The 10 highest membership ratios | | |
| Municipality | Region | *Mprc* | Municipality | Region | *Mprc* |
| Ishøj | *Cap* | 0.551 | Hjørring | *NorJ* | 0.897 |
| København | *Cap* | 0.585 | Vesthimmerland | *NorJ* | 0.899 |
| Albertslund | *Cap* | 0.588 | Assens | *South* | 0.899 |
| Brøndby | *Cap* | 0.622 | Ringkøbing-Skjern | *MidJ* | 0.900 |
| Frederiksberg | *Cap* | 0.648 | Varde | *South* | 0.902 |
| Høje-Taastrup | *Cap* | 0.678 | Nordfyns | *South* | 0.904 |
| Gladsaxe | *Cap* | 0.690 | Skive | *MidJ* | 0.905 |
| Herlev | *Cap* | 0.704 | Brønderslev | *NorJ* | 0.912 |
| Hvidovre | *Cap* | 0.708 | Læsø | *Island* | 0.916 |
| Rødovre | *Cap* | 0.711 | Lemvig | *MidJ* | 0.920 |

Note: Half the low end municipalities are the same as in Table 3, while there are two overlaps in the high end.

*2.4 The correlations between the variables*

Table 5 reports the correlations between the main variables analyzed. The discussion concentrates on rows R1 and R2. As mentioned in the introduction the analysis looks for:

(i) The links between *Lndens* and *Mprc* and the quantitative variables, which are columns C3 to C5. The two rows (R1 and R2) are very similar, and only the correlations in columns C3 to *Lnarea* and in C5 to *Lntb* are significant. The correlations to *Lnpop*, though negative as expected, are much smaller than the ones to *Lnarea*.

The correlations to the regional dummies are given in columns C6 to C11. Only the correlations to *Cap* are significant. Also, it is interesting that the correlations to the other regional dummies are small and positive. This is as suggested by Figures 3 and 4.

(ii) The link between *Lndens* and *Mprc*. It is no less than 0.82, so the two outcome variables are strongly correlated, as was already shown on Figure 2.

Table 5. Correlations between the two outcomes and the nine explanations

| | | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Outcomes | | Area, pop and income | | | | Regional dummies | | | | |
| | | *Lndens* | *Mproc* | *Lnarea* | *Lnpop* | *Lntb* | *Cap* | *Zea* | *South* | *MidJ* | *NorJ* | *Island* |
| R1 | *Lndens* | 1 | **0.82** | **0.74** | -0.28 | **-0.66** | **-0.80** | 0.13 | 0.16 | 0.26 | 0.28 | 0.27 |
| R2 | *Mproc* | **0.82** | 1 | **0.76** | -0.19 | **-0.49** | **-0.78** | 0.07 | 0.26 | 0.29 | 0.27 | 0.13 |
| R3 | *Lnarea* | **0.74** | **0.76** | 1 | **0.30** | **-0.68** | **-0.78** | 0.11 | 0.28 | **0.35** | 0.29 | -0.10 |
| R4 | *Lnpop* | -0.28 | -0.19 | **0.30** | 1 | -0.04 | -0.01 | 0.03 | 0.11 | 0.17 | 0.05 | **-0.58** |
| R5 | *Lntb* | **-0.66** | **-0.49** | **-0.68** | -0.04 | 1 | **0.69** | -0.02 | -0.25 | -0.27 | -0.27 | -0.09 |
| R6 | *Cap* | **-0.80** | **-0.78** | **-0.78** | -0.01 | **0.69** | 1 | -0.29 | -0.32 | -0.30 | -0.21 | -0.15 |
| R7 | *Zea* | 0.13 | 0.07 | 0.11 | 0.03 | -0.02 | -0.29 | 1 | -0.23 | -0.22 | -0.15 | -0.11 |
| R8 | *South* | 0.16 | 0.26 | 0.28 | 0.11 | -0.25 | -0.32 | -0.23 | 1 | -0.24 | -0.17 | -0.12 |
| R9 | *MidJ* | 0.26 | 0.29 | **0.35** | 0.17 | -0.27 | -0.30 | -0.22 | -0.24 | 1 | -0.16 | -0.11 |
| R10 | *NorJ* | 0.28 | 0.27 | 0.29 | 0.05 | -0.27 | -0.21 | -0.15 | -0.17 | -0.16 | 1 | -0.08 |
| R11 | *Island* | 0.27 | 0.13 | -0.10 | **-0.58** | -0.09 | -0.15 | -0.11 | -0.12 | -0.11 | -0.08 | 1 |

Note: As the series are non-normal, the standard significance level (0.20) for the coefficient of correlation is not used, but the coefficients from 0.30 and up are bolded. The correlations between the regional dummies make no sense so they are shaded in gray.

As explained in section 1.2 (in the introduction) the other papers in the project suggest that income, *Lntb*, and large towns, notably *Cap*, should have the same negative effect on both *Lndens* and *Mprc*. This is certainly the case.

Table 6. A confirmation using distribution free (rank) correlations

| | | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *Kendall's τ* | | | | | | *Spearman's ρ* | | | |
| | | *Lndens* | *Mproc* | *Lnarea* | *Lnpop* | *Lntb* | *Cap* | *Lndens* | *Mproc* | *Lnarea* | *Lnpop* | *Lntb* | *Cap* |
| R1 | *Lndens* | 1 | **0.65** | **0.45** | -0.16 | **-0.50** | **-0.40** | 1 | **0.84** | **0.63** | -0.22 | **-0.70** | **-0.75** |
| R2 | *Mproc* | **0.65** | 1 | **0.50** | -0.09 | **-0.49** | **-0.39** | **0.84** | 1 | **0.69** | -0.13 | **-0.71** | **-0.74** |
| R3 | *Lnarea* | **0.45** | **0.50** | 1 | **0.29** | **-0.52** | **-0.38** | **0.63** | **0.69** | 1 | **0.41** | **-0.72** | **-0.72** |
| R4 | *Lnpop* | -0.16 | -0.09 | **0.29** | 1 | -0.02 | -0.06 | -0.22 | -0.13 | **0.41** | 1 | -0.04 | -0.10 |
| R5 | *Lntb* | **-0.50** | **-0.49** | **-0.52** | -0.02 | 1 | **0.36** | **-0.70** | **-0.71** | **-0.72** | -0.04 | 1 | **0.68** |
| R6 | *Cap* | **-0.40** | **-0.39** | **-0.38** | -0.06 | **0.36** | - | **-0.75** | **-0.74** | **-0.72** | -0.10 | **0.68** | 1 |

Note: The results are the same using the original series: *Dens*, *Area*, *Pop*, *Tb*.

Table 6 is just a control of the main correlation using the two standard distribution free rank correlations. It is a well-known property of the tests that Kendall's τ is numerically smaller (by at least 50 %) than Spearman's ρ. It is reassuring that the pattern in the correlations is the same for all three correlations used. Especially, it should be noted that both outcomes have rather similar negative correlations to *Lntb* and *Cap*.

Table 7 makes two points: First, it shows that the logarithmic transformations are better than the original series in most cases. Second, it shows that *All*, which contains all churches, is slightly inferior to *Net*, which only considers parish churches. This is why *Dens* is based on *Net*, but when rows R10 and R11 are compared, it is clear that it hardly matters.

Table 7. Correlations between the quantitative variables and the auxiliary variables

| | | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Lndens* | *Mproc* | *Lnarea* | *Lnpop* | *Lntb* | *Dens* | *Area* | *Pop* | *TaxBase* | *All* | *Net* |
| R1 | *Lndens* | 1 | 0.82 | 0.74 | -0.28 | -0.66 | **(0.92)** | **0.58** | -0.21 | **-0.63** | **0.46** | **0.47** |
| R2 | *Mproc* | 0.82 | 1 | 0.76 | -0.19 | -0.49 | **0.66** | **0.60** | **-0.30** | **-0.45** | **0.34** | **0.36** |
| R3 | *Lnarea* | 0.74 | 0.76 | 1 | 0.30 | -0.68 | **0.52** | **(0.86)** | 0.10 | **-0.64** | **0.72** | **0.73** |
| R4 | *Lnpop* | -0.28 | -0.19 | 0.30 | 1 | -0.04 | **-0.43** | **0.36** | **(0.73)** | -0.04 | **0.62** | **0.62** |
| R5 | *Lntb* | -0.66 | -0.49 | -0.68 | -0.04 | 1 | **-0.56** | **-0.58** | -0.03 | **(0.99)** | **-0.51** | **-0.51** |
| R6 | *Dens* | **(0.92)** | **0.66** | **0.52** | **-0.43** | -0.56 | 1 | **0.41** | -0.25 | **-0.52** | **0.34** | **0.34** |
| R7 | *Area* | **0.58** | **0.60** | **(0.86)** | **0.36** | -0.58 | **0.41** | 1 | 0.12 | **-0.54** | **0.76** | **0.76** |
| R8 | *Pop* | -0.21 | **-0.30** | 0.10 | **(0.73)** | -0.03 | -0.25 | 0.12 | 1 | -0.04 | **0.61** | **0.59** |
| R9 | *Tb* | **-0.63** | **-0.45** | **-0.64** | -0.04 | **(0.99)** | **-0.52** | **-0.54** | -0.04 | 1 | **-0.48** | **-0.48** |
| R10 | *All* | **0.46** | **0.34** | **0.72** | **0.62** | -0.51 | **0.34** | **0.76** | **0.61** | **-0.48** | 1 | **1.00** |
| R11 | *Net* | **0.47** | **0.36** | **0.73** | **0.62** | -0.51 | **0.34** | **0.76** | **0.59** | **-0.48** | **1.00** | 1 |

Note: See note to Table 5. The correlations in the top square to the left are shaded in gray as they are the same as the corresponding ones in Table 5. The correlations in brackets are the variables correlated with their own logarithmic transformation.

## 3. The regressions

The regressions use the following framework:

(1a) $$Ldens_i = \beta_{1d} Lnarea_i + \beta_{2d} Lnpop_i + \beta_{3d} Lntb + \sum_{i=1}^{5} \gamma_{di} reg_i + Con_d + u_{id}$$

(1b) $$Mprc_i = \beta_{1m} Lnarea_i + \beta_{2m} Lnpop_i + \beta_{3m} Lntb + \sum_{i=1}^{5} \gamma_{mi} reg_i + Con_m + u_{id}$$

where *reg* = *Cap*, *Zea*, *NorJ*, *South* and *Island*, i.e., *MidJ* is deleted as the most typical region taken as the constant. *Con* is the constant. The standard testing down method gives the following two relations:

(2a) $$Ldens_i = \beta_{1d} Lnarea_i + \beta_{2d} Lnpop_i + \gamma_d Cap_i + Con_d + u_{id}$$

(2b) $$\begin{aligned} Mprc_i = {} & \beta_{1m} Lnarea_i + \beta_{2m} Lnpop_i + \gamma_{1m} Cap_i \\ & + \beta_{3m} Lntb + \gamma_{2m} Zea_i + \gamma_{3m} Island_i + Con_m + u_{im} \end{aligned}$$

The two equations are fairly parallel. They are estimated by OLS, and all coefficients are significant – also if robust standard errors are used. Two additional relations are discussed:

(3a) $$Ldens_i = \beta_{1d} Lnarea_i + \beta_{2d} Lnpop_i + \gamma_d Lntb_i + Con_d + u_{id}$$

(3b) $$Mprc_i = \beta_{1m} Lnarea_i + \beta_{2m} Lnpop_i + \gamma_{1m} Cap_i + Con_m + u_{im}$$

### *3.1 Explaining Lndens, the logarithmic church density*

Table 8 contains the estimates of equations (1a) and (2a) that are reached by testing down (1a). A comparison of equations (2a) and (3a) shows that *Cap* and *Lntb* (Income) are almost equally good as an explanatory variable. However, it should be noted from equation (7a) that *Lnpop* and *Lnarea* provide most of the explanatory power.

Equations (4a) to (12a) are variants showing the effects of the individual variables. It is interesting to look at (3a) to (5a) showing that while *Lnarea* explains much more than *Lnpop* when they are the only regressor, *Lnpop* adds substantially when both variables are in together. It is also interesting to compare (2a) and (7a) showing the pattern of multicollinearity between *Lnarea*, *Lnpop* and *Cap* (the capital region). There is some multicollinearity, but all three variables clearly belong. However, all the 5 remaining regional variables are fully represented by *Lnarea* and *Lnpop*.

Table 8. Regressions explaining *Lndens*

| Eq. | *Lnarea* | *Lnpop* | *Lntb* | *Cap* | *Zea* | *MidJ* | *NorJ* | *South* | *Island* | *Con* | $R^2$ *adj* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1a) | **0.36** | **-0.46** | -0.41 | **-0.50** | -0.05 | | 0.08 | -0.15 | 0.01 | **9.47** | 0.85 |
| | (7.9) | (-8.8) | (-1.3) | (-3.3) | (-0.4) | | (0.6) | (-1.5) | (0.1) | (2.4) | |
| (2a) | **0.39** | **-0.48** | | **-0.48** | | | | | | **4.56** | 0.85 |
| | (9.2) | (-11) | | (-4.0) | | | | | | (10.7) | |
| (3a) | **0.47** | **-0.53** | **-0.75** | | | | | | | **13.46** | 0.83 |
| | (13) | (-12) | (-2.4) | | | | | | | (3.5) | |
| (4a) | **0.43** | | | | | | | | | **-0.95** | 0.55 |
| | (11) | | | | | | | | | (-4.2) | |
| (5a) | | **-0.28** | | | | | | | | **4.35** | 0.07 |
| | | (-2.8) | | | | | | | | (4.2) | |
| (6a) | | | **-3.57** | | | | | | | **44.28** | 0.43 |
| | | | (-8.7) | | | | | | | (9.0) | |
| (7a) | **0.53** | **-0.55** | | | | | | | | **4.41** | 0.82 |
| | (20) | (-12) | | | | | | | | (9.7) | |
| (8a) | **0.53** | **-0.55** | | | 0.10 | | | | | **4.40** | 0.82 |
| | (20) | (-12) | | | (1.1) | | | | | (9.7 | |
| (9a) | **0.52** | **-0.56** | | | | 0.09 | | | | **4.47** | 0.82 |
| | (19) | (-12) | | | | (1.0) | | | | (9.7) | |
| (10a) | **0.52** | **-0.55** | | | | | 0.14 | | | **4.42** | 0.82 |
| | (19) | (-12) | | | | | (1.2) | | | (9.7) | |
| (11a) | **0.54** | **-0.55** | | | | | | -0.08 | | **4.38** | 0.82 |
| | (20) | (-12) | | | | | | (-0.9) | | (9.6) | |
| (12a) | **0.53** | **-0.52** | | | | | | | 0.20 | **4.04** | 0.82 |
| | (20) | (-9.4) | | | | | | | (1.1) | (7.1) | |

Note also to Table 9: The main regression is Eq (2a) that it is reached when regression (1a) is tested down. Significant coefficients are bolded. *N* = 98 in all regressions.

The main regression (2a) is rather satisfactory with both high t-ratios and a fine $R^2$-score. It is possible to polish the relation by deleting/including some borderline cases (such as the ones listed in Appendix table A4) in *Cap*, but it barely matters for the result.

**Interpretation of the sizes of the effects in (2b):** As all quantitative variables are in logs on both sides, the coefficients are *elasticities*. If the municipality is compared with one with twice the area, but the same number of people, it has 39 % more churches. If a municipality is compared with one with twice as many people, but the same area, the density of churches falls by 48 %. The two elasticities add to –0.1, so a municipality with twice the area and population has slightly less than twice the number of churches. The *Cap*-variable is not in logs, so the results show that the municipalities in the Copenhagen region have $e^{0.5} \approx$ 1.6 less churches per 10'000 inhabitants, much as suggested by Figure 3.

### 3.2 *Explaining Mprc, the church membership ratio*

Table 9 explains the membership ratio. The variation is much less, the scaling differs, and it is not in logs. Consequently, all coefficients are about 10 times smaller in Table 9 than in Table 8, and hence one more digit is reported in Table 9.

Table 9. Regressions explaining *Mprc*

| Eq. | *Lnarea* | *Lnpop* | *Lntb* | *Cap* | *Zea* | *MidJ* | *NorJ* | *South* | *Island* | *Con* | $R^2$ adj |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1b) | **0.042** | **-0.047** | **0.164** | **-0.084** | **-0.030** | | 0.007 | -0.002 | **-0.049** | **-0.838** | 0.83 |
| | (8.8) | (-8.6) | (4.9) | (-5.2) | (-2.7) | | (0.5) | (-0.2) | (-2.4) | (-2.1) | |
| (2b) | **0.043** | **-0.048** | **0.162** | **-0.083** | **-0.031** | | | | **-0.049** | **-0.818** | 0.84 |
| | (9.1) | (-8.8) | (4.9) | (-5.7) | (-3.2) | | | | (-2.6) | (-2.0) | |
| (3b) | **0.040** | **-0.040** | | **-0.041** | | | | | | **1.041** | 0.78 |
| | (8.0) | (-7.4) | | (-2.9) | | | | | | (20.6) | |
| (4b) | **0.044** | | | | | | | | | **0.585** | 0.57 |
| | (11) | | | | | | | | | (27) | |
| (5b) | | ***-0.019*** | | | | | | | | **1.022** | 0.03 |
| | | (-1.9) | | | | | | | | (9.6) | |
| (6b) | | | **-0.263** | | | | | | | **3.985** | 0.24 |
| | | | (-5.6) | | | | | | | (7.0) | |
| (7b) | **0.060** | **-0.049** | **0.101** | | | | | | | -0.193 | 0.78 |
| | (15) | (-9.7) | (2.9) | | | | | | | -0.5 | |
| (8b) | **0.046** | **-0.042** | **0.144** | **-0.059** | | | | | | -0.695 | 0.81 |
| | (9.5) | (-8.5) | (4.2) | (-4.2) | | | | | | (-1.7) | |
| (9b) | **0.060** | **-0.049** | **0.103** | | -0.006 | | | | | -0.215 | 0.78 |
| | (15) | (-9.6) | (2.9) | | (-0.6) | | | | | (-0.5) | |
| (10b) | **0.059** | **-0.050** | **0.103** | | | 0.014 | | | | -0.202 | 0.78 |
| | (14) | (-9.8) | (2.9) | | | (1.3) | | | | (-0.5) | |
| (11b) | **0.059** | **-0.049** | **0.105** | | | | 0.013 | | | -0.237 | 0.78 |
| | (15) | (-9.6) | (2.9) | | | | (1.0) | | | (-0.6) | |
| (12b) | **0.059** | **-0.050** | **0.108** | | | | | 0.016 | | -0.272 | 0.79 |
| | (15) | (-9.8) | (3.1) | | | | | (1.6) | | (-0.6) | |
| (13b) | **0.060** | **-0.053** | **0.097** | | | | | | -0.023 | -0.105 | 0.78 |
| | (15) | (-8.6) | (2.7) | | | | | | (-1.1) | (-0.2) | |
| (14b) | | | | | 0.017 | | | | 0.048 | **0.819** | 0.00 |
| | | | | | (0.8) | | | | (1.3) | (91) | |

Note: If *Lntb* is replaced by *Tb*, it has virtually no effect.

If the difference in scaling is disregarded, the pattern reported in the two tables is rather similar, except for one problem: When income, *Lntb*, is included it gets a positive sign and the effect of *Cap* becomes twice as large. The positive sign on *Lntb* is contrary to the theory discussed in section 1.2 and the correlations in Tables 5 and 6 and regression (6b). Also, two regional dummies, *Zea* and *Iland*, become significant and obtain negative signs while they should have positive signs. The positive signs follow from the correlations in Table 5 and from equations (9b) and (12b), not to mention regression (14b). These sign-shifts point to multicollinearity. The equation has too many highly correlated variables.

This argues for using the estimate (3b) that is parallel to (2a) and where all estimated coefficients make sense, also the relative sizes of the effects in (3b) and (2a) are rather similar. If income is deleted, the testing down version becomes (3b) which is the same as (2a).

**Interpretation of the sizes of the effects in (3b):** The two quantitative explanations are in logs and of the same numerical size, and as ln(2) ≈ 0.69 and 0.040·0.69 ≈ 3. Hence, a crude indication of the results is that if the municipality is compared with one with twice the area, but the same number of people, church membership is 3 percentage points larger. If a municipality is compared with one with twice as many people, but the same area, church membership is 3 percentage points less. It further follows that church membership is 4 percentage points lower in the capital region when corrected for geographical differences.

### *3.3 The interaction of Lndens and Mprc*

Table 5 reported a high correlation (0.82) between the two endogenous variables *Lndens* and *Mprc*. They are explained rather well by the same variables in roughly the same proportions, so they must be correlated, but it is interesting if they are related in any other way.

Table 10. Adding the other explained variable

| Eq. | Explaining *Lndens* | | | | | | Eq. | Explaining *Mprc* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Mproc* | *Lnarea* | *Lnpop* | *Cap* | *Con* | $R^2$ adj | | Lndens | *Lnarea* | *Lnpop* | *Cap* | *Con* | $R^2$ adj |
| (13a) | **8.27** | | | | **-5.41** | 0.67 | (15b) | **0.082** | | | | **0.709** | 0.67 |
| | (14) | | | | (-11) | | | (14) | | | | (75) | |
| (14a) | 1.16 | **0.47** | **-0.50** | | **3.22** | 0.83 | (16b) | 0.015 | **0.044** | **-0.037** | | **0.961** | 0.77 |
| | (1.3) | (8.8) | (-8.3) | | (3.1) | | | (1.3) | (6.4) | (-4.5) | | (13) | |
| (15a) | 0.25 | **0.38** | **-0.47** | **-0.47** | **4.30** | 0.85 | (17b) | 0.003 | **0.039** | **-0.038** | **-0.040** | **1.025** | 0.78 |
| | (0.3) | (6.9) | (-8.3) | (-3.7) | (4.3) | | | (0.3) | (5.6) | (-4.8) | (-2.5) | (13) | |

Table 10 shows that they are not – using (2a) and (3b). Once the relation is controlled for the three variables in the main model, *Mprc* adds nothing to the *Lndens* relation and vice versa. The same result is reached when (3b) is replaced with (2b) explaining.

This completes the regression study of the causal structure as shown on Figure 1, but section 4 returns to the interplay between income and the capital region.

*3.4 The cross-regional and cross-municipal stability of the two main relations*

A simple way to control the relations for stability is to systematically delete municipalities and see if the relation changes. This turns out to be a rather dull enterprise as the two relations barely change, so this exercise will not be reported. Table 11 shows a stronger experiment, where each region is deleted. The table is in two sections:

The first section looks at the estimate of *Lndens*, i.e., Equation (1a) in the table is the same as (2a) in Table 7. Then the equation is re-estimated after the deletion of each region one at the time. When the *Cap*-region is deleted the *Cap*-dummy is irrelevant, so row (2a) in the table is a bit different from the other ones. However, even row (2a) does not differ much. The second half of the table repeats the same exercise for the main relation explaining *Mprc*, showing parallel results.

Column (6) is shaded in gray. It reports estimates of the same relation adding the other explanatory variable, but as the results to the three main variables change marginally they are not reported. Only the coefficients to the new variable are reported. None of these estimates are even approaching significance.

Table 11. The robustness of the main equations to deletion of regions

| | | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|---|
| | | Estimates explaining *Lndens* using equation (2a) | | | | | |
| | | *Lnarea* | *Lnpop* | *Cap* | Constant | $R^2$ adj | *Mproc* added |
| (2a) | All | **0.391** (9.2) | **-0.484** (-11) | **-0.481** (-4.0) | **4.557** (11) | 0.847 | 0.246 (0.3) |
| (2.1a) | No Cap | **0.518** (10) | **-0.650** (-14) | | **5.544** (14) | 0.741 | 1.304 (0.9) |
| (2.2a) | No Zea | **0.335** (6.7) | **-0.452** (-9.3) | **-0.625** (-4.3) | **4.574** (10) | 0.854 | 0.480 (0.5) |
| (2.3a) | No MidJ | **0.385** (8.3) | **-0.467** (-9.2) | **-0.481** (-3.7) | **4.403** (9.2) | 0.846 | -0.206 (-0.2) |
| (2.4a) | No NorJ | **0.375** (8.7) | **-0.461** (-10) | **-0.495** (-4.1) | **4.383** (10) | 0.849 | 0.428 (0.5) |
| (2.5a) | No South | **0.389** (8.6) | **-0.442** (-9.0) | **-0.529** (-4.1) | **4.165** (9.2) | 0.866 | 0.032 (0.0) |
| (2.6a) | No Island | **0.389** (9.0) | **-0.496** (-9.4) | **-0.492** (-4.0) | **4.702** (8.8) | 0.844 | 0.124 (0.1) |
| | | Estimates explaining *Mprc* using equation (3b) | | | | | |
| | | *Lnarea* | *Lnpop* | *Cap* | Constant | $R^2$ adj | *Lndens* added |
| (3b) | All | **0.040** (8.0) | **-0.040** (-7.4) | **-0.041** (-2.9) | **1.041** (21) | 0.782 | 0.003 (0.3) |
| (3.1b) | No Cap | **0.036** (8.2) | **-0.035** (-8.7) | | **1.019** (30) | 0.562 | 0.010 (0.9) |
| (3.2b) | No Zea | **0.042** (6.9) | **-0.040** (-6.6) | **-0.040** (-2.3) | **1.031** (19) | 0.792 | 0.007 (0.5) |
| (3.3b) | No MidJ | **0.039** (6.8) | **-0.038** (-6.1) | **-0.042** (-2.6) | **1.026** (18) | 0.763 | -0.003 (-0.2) |
| (3.4b) | No NorJ | **0.039** (7.3) | **-0.040** (-6.9) | **-0.041** (-2.7) | **1.045** (19) | 0.768 | 0.007 (0.5) |
| (3.5b) | No South | **0.043** (7.3) | **-0.041** (-6.4) | **-0.033** (-2.0) | **1.031** (18) | 0.775 | 0.001 (0.0) |
| (3.6b) | No Island | **0.040** (7.7) | **-0.042** (-6.7) | **-0.043** (-2.9) | **1.069** (17) | 0.779 | 0.002 (0.1) |

Note: Column (6) is the same regression as in (1) to (5), but with the extra regressor added. As it is nowhere significant the rest of the regression is virtually the same as in columns (1) to (5).

### *3.5 The logarithmic transformation of Dens*

The reader may want to see how the logarithmic transformation of *Dens*, *Area* and *Pop* affects the results. In small samples this is often crucial, but as the analysis considers the sample of 98 municipalities, it does not matter for the qualitative structure in the results, but it does influence the fit substantially. This is shown in Table 12.

Table 12. Explaining *Dens* and *Lndens*, *N* = 98

| Plain data | | The dependent variable is *Dens* | | | | |
|---|---|---|---|---|---|---|
| (Not logs) | | *Area* | *Pop* | *Cap* | *Cons* | $R^2$ adj |
| | (1) | | **-0.15** (-2.6) | | **6.24** (13) | 0.05 |
| | (2) | **0.41** (4.4) | | | **3.58** (6.6) | 0.16 |
| | (3) | **0.45** (5.0) | **-0.18** (-3.4) | | **4.44** (7.7) | 0.24 |
| | (4) | **0.08** (0.8) | **-0.14** (-3.2) | **-4.80** (-5.8) | **7.24** (11) | 0.44 |
| Log version | | The dependent variable is *Dens* | | | | |
| | | *Lnarea* | *Lnpop* | *Cap* | *Cons* | $R^2$ adj |
| | (5) | | **-2.05** (-4.6) | | **27.19** (5.8) | 0.17 |
| | (6) | **1.46** (6.0) | | | ***-2.59*** (-1.9) | 0.27 |
| | (7) | **2.01** (11.4) | **-3.10** (-10) | | **27.43** (8.9) | 0.65 |
| | (8) | **1.51** (5.0) | **-2.85** (-8.8) | **-1.75** (-2.0) | **27.97** (9.2) | 0.66 |
| Log-log version | | The dependent variable is *Lndens* | | | | |
| | | *Lnarea* | *Lnpop* | *Cap* | *Cons* | $R^2$ adj |
| | (9) | | **-0.28** (-2.8) | | **4.35** (4.2) | 0.07 |
| | (10) | **0.43** (10.8) | | | **-0.95** (-4.2) | 0.55 |
| | (11) | **0.53** (20.3) | **-0.55** (-12) | | **4.41** (9.7) | 0.82 |
| **Main** | (12) | **0.39** (9.2) | **-0.48** (-11) | **-0.48** (-4.0) | **4.56** (11) | 0.85 |

Note: All regressions in the last section of the table were also reported in Table 8 above.

# 4. Another look at income versus capital region

To further explore the relationship between the two related explanatory variables *Lntb* and *Cap*, they are shown in scatter diagrams of *Lndens* and *Mprc* over *Lntb*.

### *4.1 Church density and the two explanations*

Figure 6 gives a fairly clear picture where the scatter is dominated by a downward sloping line, and as we already know, nearly all the *Cap*-points are to the right on the figure.

Figure 6. Scatter of *Lndens* over *Lntb*, income, with *Cap* marked

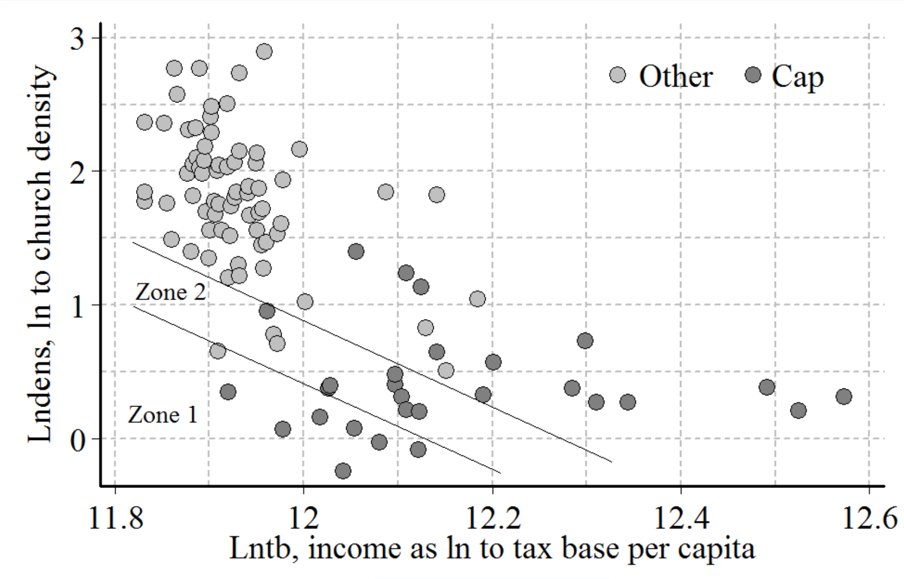



Table 13. The municipalities in the two zones marked on Figure 6

| Zone 1: The most deviating cases | | | | Zone 2: The moderately deviating cases | | | |
|---|---|---|---|---|---|---|---|
| Municipality | Region | *Lntb* | *Lndens* | Municipality | Region | *Lntb* | *Lndens* |
| Odense | South | 11.909 | 0.657 | Halsnæs | Cap 2 | 11.961 | 0.952 |
| Ishøj | Cap 1 | 11.920 | 0.351 | Fredericia | South | 11.968 | 0.784 |
| Albertslund | Cap 3 | 11.978 | 0.073 | Aarhus | MidJ | 11.972 | 0.708 |
| Brøndby | Cap 4 | 12.018 | 0.160 | Høje-Taastrup | Cap 5 | 12.026 | 0.375 |
| Hvidovre | Cap 7 | 12.042 | -0.240 | København | Cap 6 | 12.029 | 0.397 |
| Rødovre | Cap 8 | 12.054 | 0.080 | Herlev | Cap 11 | 12.097 | 0.408 |
| Tårnby | Cap 10 | 12.081 | -0.029 | Helsingør | Cap 12 | 12.098 | 0.485 |
| Glostrup | Cap 16 | 12.122 | -0.080 | Vallensbæk | Cap 13 | 12.104 | 0.315 |
| | | | | Ballerup | Cap 14 | 12.109 | 0.221 |
| | | | | Gladsaxe | Cap 17 | 12.123 | 0.200 |

Note: The number given after *Cap* is the income rank from 1for the lowest to 28 for the highest.

The observations that are most contrary to this picture are the ones in Zone 1 and less so the ones in Zone 2. Table 13 shows that two zones contain the poorer *Cap* municipalities.

## *4.2 Church membership and the two explanations*

When the same analysis is made for *Mprc* the picture of the point scatter on Figure 7 is more interesting: The negative slope is still clear, but now Zone 1 deviates rather strongly. It consists of the 5 poorest municipalities in the capital region.[3]

Figure 7. Scatter of *Mprc* over *Lntb*, income, with *Cap* marked

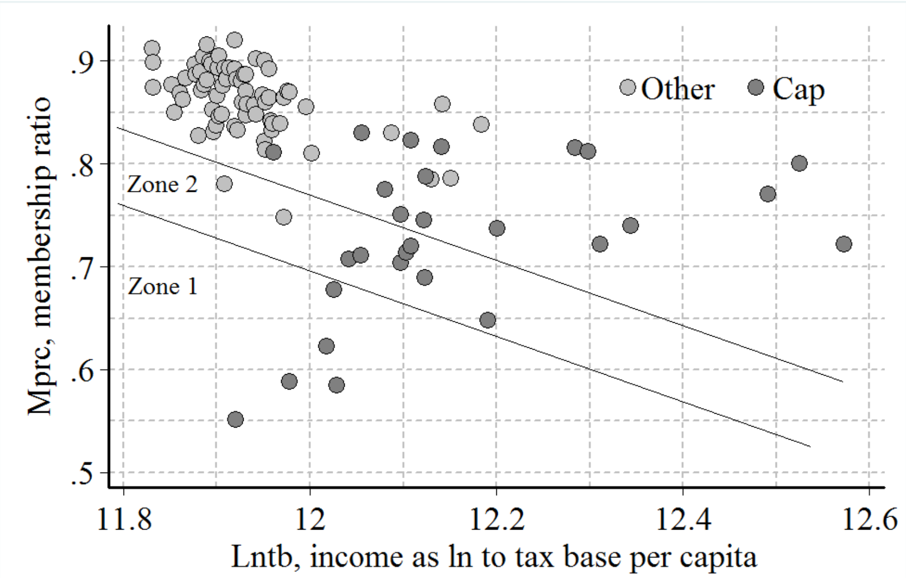



Table 14. The municipalities in the two zones marked on Figure 7

| Zone 1: The most deviating cases | | | | Zone 2: The moderately deviating cases | | | |
|---|---|---|---|---|---|---|---|
| Municipality | Region | *Lntb* | *Lndens* | Municipality | Region | *Lntb* | *Lndens* |
| Ishøj | Cap 1 | 11.920 | 0.551 | Hvidovre | Cap 7 | 12.042 | 0.708 |
| Albertslund | Cap 3 | 11.978 | 0.588 | Rødovre | Cap 8 | 12.054 | 0.711 |
| Brøndby | Cap 4 | 12.018 | 0.622 | Herlev | Cap 11 | 12.097 | 0.704 |
| Høje-Taastrup | Cap 5 | 12.026 | 0.678 | Vallensbæk | Cap 13 | 12.104 | 0.714 |
| København | Cap 6 | 12.029 | 0.585 | Ballerup | Cap 14 | 12.109 | 0.720 |
| | | | | Gladsaxe | Cap 17 | 12.123 | 0.690 |
| | | | | Frederiksberg | Cap 20 | 12.191 | 0.648 |
| | | | | Odense | South | 11.909 | 0.780 |
| | | | | Aarhus | MidJ | 11.972 | 0.748 |

Note: see note to Table 13.

3. One of the 'poor' municipality in the Cap region appears outside the two zones. It is Halsnæs (around Fredriksværk) which is not a truly 'metropolitan' area.

The 5 municipalities in Zone 1 are the ‘labor’ suburbs and Copenhagen itself, and they have large immigrant populations. So the picture does make a great deal of sense.

## 5. Population density and the ideal number of services

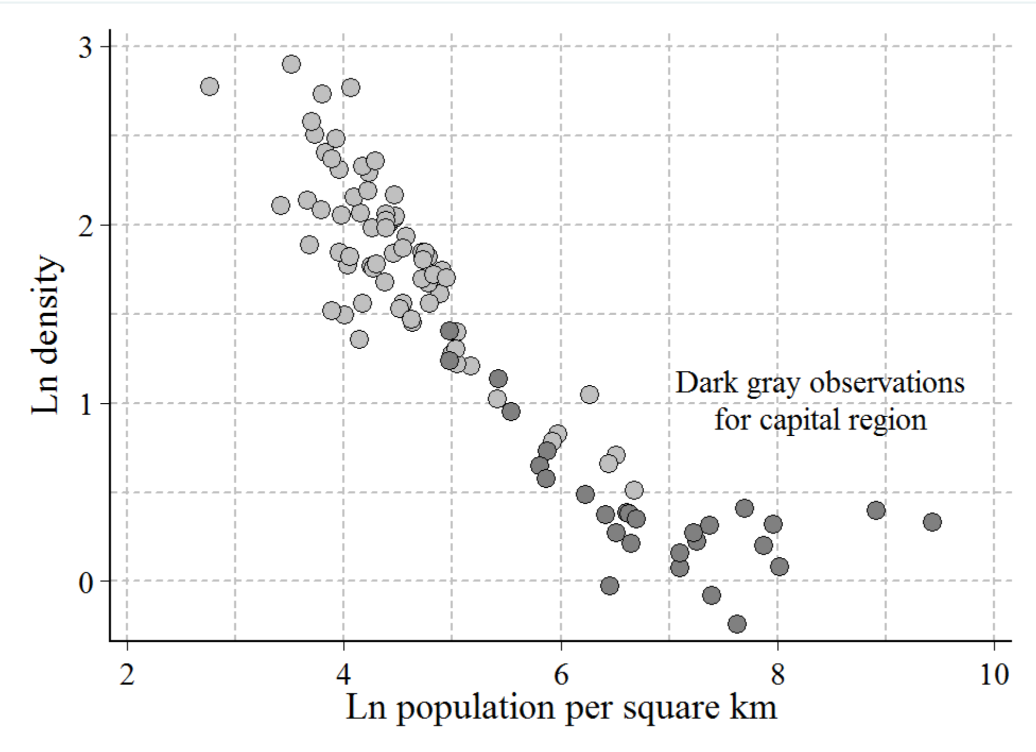
The last paragraphs of sections 3.1 and 3.2 convert the regressions results to effect sizes. This is perhaps the best ways to understand the results, but two more techniques will be used.

### *5.1 Population densities*

The regressions separate out the effects of area and population, but as the effects are roughly similar numerically it is possible to combine the two explanations in the form a population density (population per km$^2$) as done in Figure 8.

The figure uses the log to population density to 'explain' *Lndens*. It is nice to see that a fairly linear path emerges on the graph. However, also some of the observations from the Capital Region are to the right of the line through the non-capital observations. The two extreme high population densities are Frederiksberg and Copenhagen, which are the two most built-up municipalities in Denmark. It is clear (also) from Figure 8 why the best regression above gives a rather satisfactory explanation from the point of view of statistics.

Figure 8. Logarithmic density 'explained' by logarithm to population density




Note: The two extreme high population densities are Frederiksberg and Copenhagen.

The next question is if the log-log regression with population and area and the Capital Region makes sense. Why should the best regression fit so well? The double log form indicates big effects of distance and size, but the intuition suffers a bit when both axes are in logs.

Figure 9a. Looking at the plain data – low population density

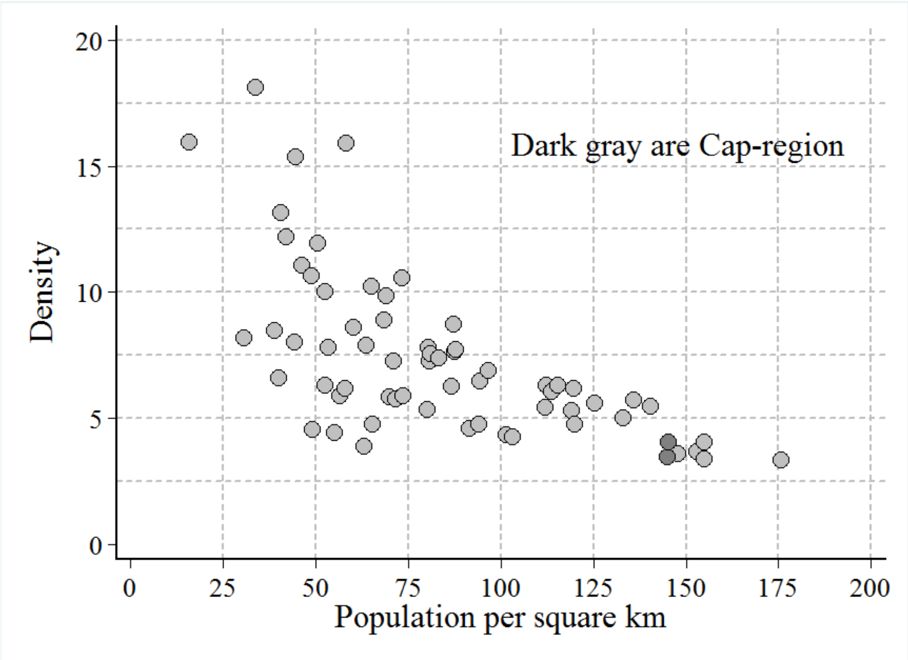
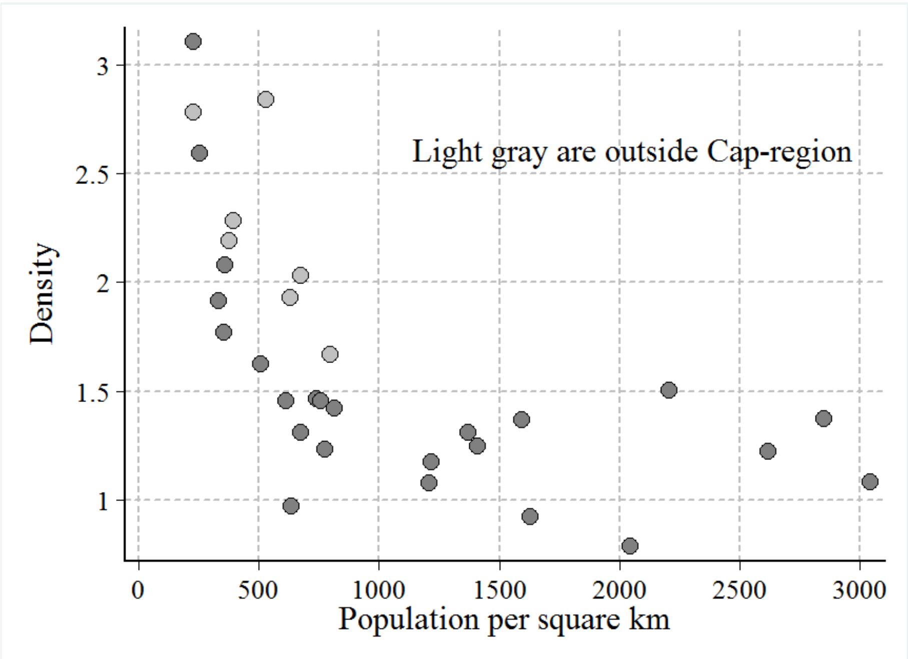



Figure 9b. Looking at the plain data – high population density



Consequently, Figure 8 has been recalculated in the plain data. To be readable it has been broken into two, and the two extreme points for Frederiksberg and Copenhagen are deleted. This has produced Figures 9a and 9b.

The average curve is assessed at five points in the small Table 15. In a suburban detached housing area with 500 inhabitants per $km^2$ there are 2 churches per 10,000 inhabitants, while there are 10 in a rural area with 50 inhabitants per $km^2$.

Virtually all rural households and all vicars have cars in Denmark, and the network of roads is dense and fine. Consequently, distances measured in time and money have shrunk radically. This has had a large effect on the density of schools and supermarkets, but a small effect on the distribution of churches.

Table 15. Five examples assessed from Figures 9a and b

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| Pop/$km^2$ | 25 | 50 | 100 | 250 | 500 |
| Church density | 17 | 10 | 6 | 3 | 2 |

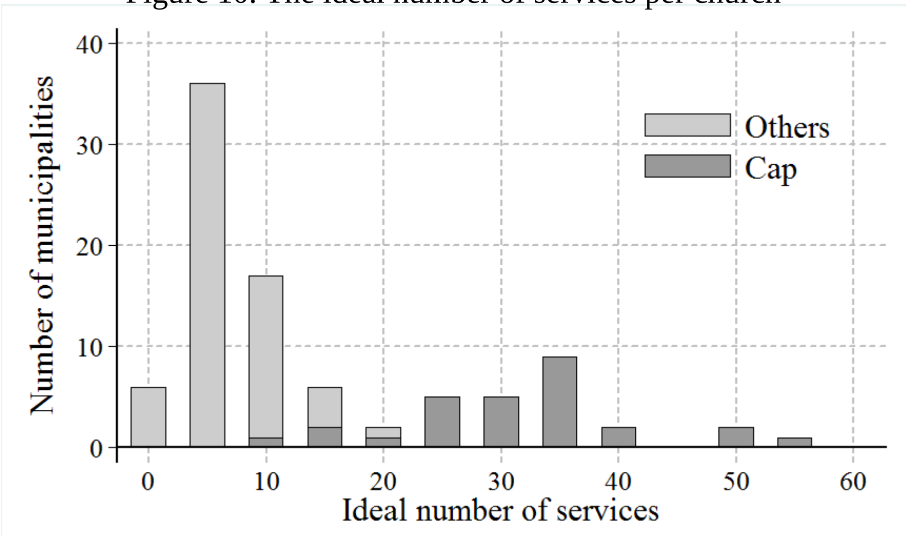
### *5.2 The 'ideal' number of services*

Another way to visualize the data is to assume that a good Lutheran should attend one service per week, preferably on Sunday. Assume further that all churches has space for 150 churchgoers.

From these assumptions it can be calculated how many services the average church in each municipality should ideally hold. The average municipality in this regard is Holbæk, which has 33 churches and 69,354 inhabitants of which 57,005 are church members. This gives 1,727 members per church, so it should ideally hold 12 services per week to accommodate all. In fact, the average church in Holbæk municipality holds less than one service per week and it is not very crowded at that occasion, giving a service attendance rate of 2-3%.

Figure 10 shows the distribution of the ideal number of services in the municipalities, and as before the municipalities with the highest and lowest scores are given in a table. The figure looks as expected with the *Cap*-region in one end – having many church members per church, and thinly populated municipalities far from the capital in the other end. The key observation from Figure 10 is the cross-country range of the numbers calculated. In the capital region the average is 34, while it is 11 in the rest of the country.

Figure 10. The ideal number of services per church




Note: Calculated as the number of services (for 150 church members) necessary to accommodate all members one time per week in the average church in the municipality.

Table 16. The extreme scores for the ideal number of services

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| The 10 highest scores | | | The 10 lowest scores | | |
| Municipality | Region | *Numbe* | Municipality | Region | *Mprc* |
| Hvidovre | *Cap* | 60 | Samsø | *Island* | 3 |
| Glostrup | *Cap* | 54 | Morsø | *NorJ* | 4 |
| Tårnby | *Cap* | 53 | Læsø | *Island* | 4 |
| Rødovre | *Cap* | 44 | Langeland | *South* | 4 |
| Hørsholm | *Cap* | 43 | Thisted | *NorJ* | 4 |
| Ballerup | *Cap* | 38 | Lolland | *Zea* | 5 |
| Gladsaxe | *Cap* | 38 | Lemvig | *MidJ* | 5 |
| Lyngby- | *Cap* | 38 | Rebild | *NorJ* | 5 |
| Dragør | *Cap* | 37 | Ærø | *Island* | 6 |
| Furesø | *Cap* | 37 | Vesthimmerland | *NorJ* | 6 |

Hvidovre (*Cap*, *Pop* = 50,871) tops the list with 9,000 church members per church, so the churches would need 60 services per week to accommodate all members. Here churches actually hold one service per week, and these services are not crowded. Samsø (*Island*, *Pop* = 3,867) has 7 churches, but only three vicars and the services are spread out, so that each vicar holds (at most) one service per week. Thus, the church use is adjusted somewhat to the densities by having fewer services than in the high density churches.

## 6. The church membership ratio – a look back and ahead

The data for church membership goes back to 1974, they are reported in Table 17. There is a small data break in 1990, but here both data exists. Figure 11 show these data.

Table 17. The national membership data

| Paying church tax | | In CPR register | |
|---|---|---|---|
| Year | Percent | Year | Percent |
| 1974 | 95.2 | 1990 | 89.3 |
| 1975 | 94.9 | 1991 | 88.9 |
| 1976 | 94.8 | 1992 | 88.2 |
| 1977 | 94.5 | 1993 | 87.7 |
| 1978 | 94.4 | 1994 | 87.4 |
| 1979 | 94.1 | 1995 | 87.0 |
| 1980 | 93.8 | 1996 | 86.5 |
| 1981 | 93.6 | 1997 | 86.1 |
| 1982 | 93.0 | 1998 | 85.8 |
| 1983 | 92.6 | 1999 | 85.4 |
| 1984 | 92.3 | 2000 | 85.1 |
| 1985 | 91.9 | 2001 | 84.7 |
| 1986 | 91.5 | 2002 | 84.3 |
| 1987 | 91.1 | 2003 | 83.8 |
| 1988 | 90.7 | 2004 | 83.4 |
| 1989 | 90.2 | 2005 | 83.3 |
| 1990 | 89.9 | 2006 | 83.0 |
| | | 2007 | 82.6 |
| | | 2008 | 82.1 |
| | | 2009 | 81.5 |
| | | 2010 | 80.9 |
| | | 2011 | 80.4 |
| | | 2012 | 79.8 |
| | | 2013 | 79.1 |

Source: Statistics Denmark various publications.

The data has a clear trend. The series is upward limited at 100 and it has never been higher than 98, as there are about 1 % Catholics, various "Free" churches, Jews, etc. that goes way back. Also there has, since the start of the $20^{th}$ century been atheists. Thus, as the series is projected backwards it has to converge to 98. It is clear that the series bends, and as immigration of non-members of the church started in the mid 1960s it is reasonable to imagine that the stylized picture is as drawn on Figure 12.

Figure 11. The national membership data

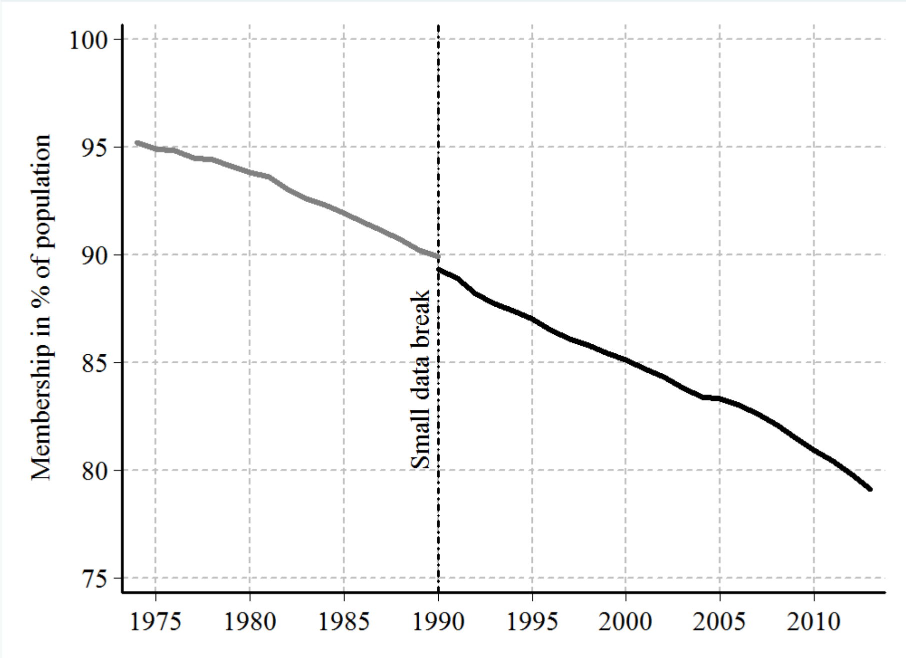
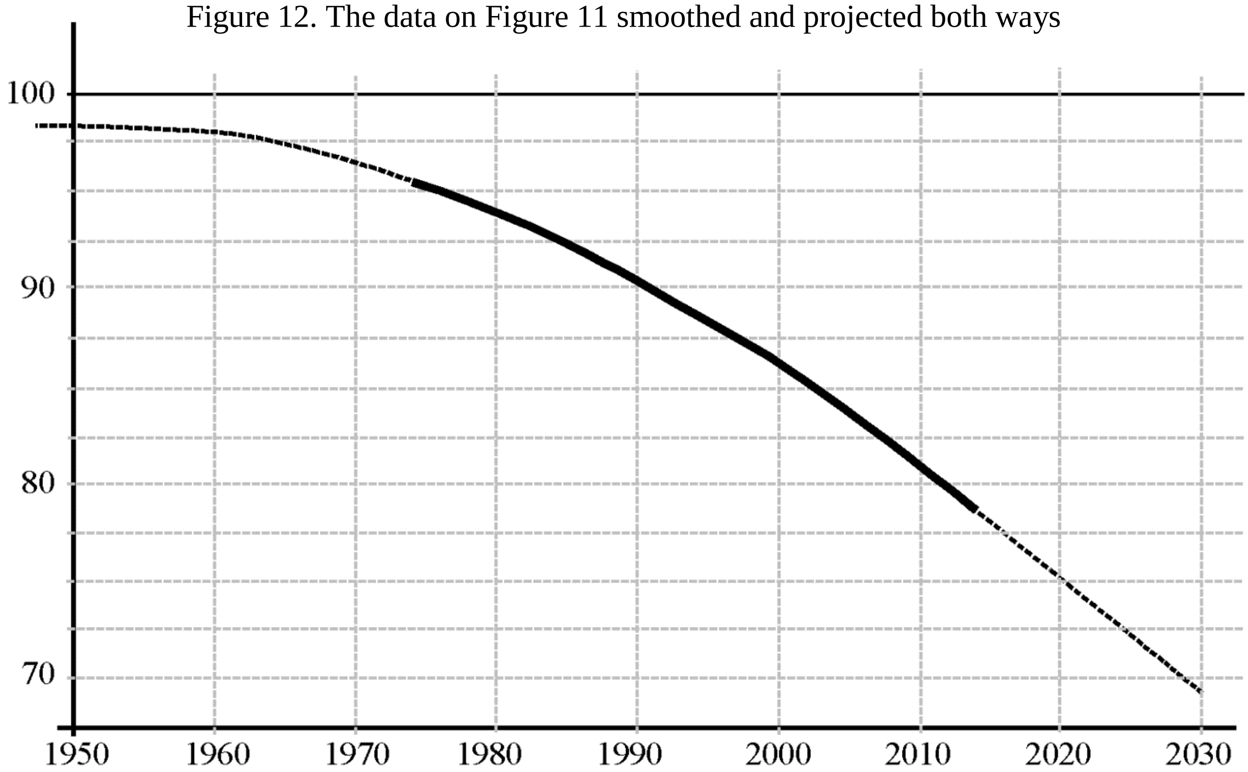



Figure 12. The data on Figure 11 smoothed and projected both ways




The black line is a stylized version of the one on Figure 11 and the broken line is the projection. The main problem generalizing and projecting the line is that it is fairly flat between 1980 and 2000, but bends at the ends. The theory of the religious transition predicts that there should be convergence *both* to the high old level and to a new lower level.

At the old end things are clear. Due to the 98 % limit it is obvious where the upper level is, and the curve on Figure 11 does bend, so here things are rather clear. The old level is thus taken to be 98 % and it was the stable long-run level before 1950, and the fall from 1950 to 1960 was still negligible.

At the new end things are less clear. There is no sign of a convergence to a new level. The membership rate is still rather high, and it looks as if the fall is (still) increasing. So I presume that the stable low level is still rather distant. But as our theory predicts that the fall will eventually level off I predict a constant fall of about 6 percentage point for the next couple of decades. Thus it looks as if a membership rate of 70 % will be reached before 2030.

It the development continues the rate will reach 50% around 2060. That will, of course make the 'Danish solution' that combines religious freedom with one 'official' church a dubious construction.

## 7. Conclusions

The cross-municipal pattern in both church density and church membership is explained rather well by a simple model using three explanatory variables: The area and population of the municipality and a binary dummy for the capital region that works very much as income in the relation. The two explained variables have no further interaction than the one generated by the three explanatory variables. This is the causal structure shown on Figure 1 in the introduction. These findings are parts of a pattern analyzed in the other papers of the project (see references). Both the cross-country and the long-run time-series studies find a strong religious transition: when countries become wealthy religiosity falls. The long-run study analyzes the aggregate church density variable:

It analyzes the *puzzle of the missing church overcrowding* in Europe. Travelers will surely have been impressed by the high age of most churches. The rather few 'new' churches often replace old ones destroyed by war or fire. Paldam *et al.* (2012a and b) will show that this impression is true.[4] The increase in the number of churches the last couple of centuries is small in Western Europe, where wealth increased dramatically. In the period the demographic transition increased the European population about five times. If people used churches as much today as they did before, they would be bursting from overcrowding, but they are not. Consequently, there must have been a large drop in religiosity.

The present paper looks at the same phenomenon by considering the cross-municipal density of churches in Denmark 2011-12. An important aspect of the puzzle is that few churches have been in the last quarter century Also, most churches belong to the national heritage, which is carefully maintained.

In the last couple of centuries the population in the villages and smaller towns has stagnated with no reduction in the number of churches. Population growth has taken place in the larger towns, especially in the Capital Region, where tiny villages have grown into large suburbs. Here few churches have been built, so church densities are low. And, it appears that churches are not missing.

This means that if church densities are used as a proxy for religiosity, the numbers underestimate the fall in religiosity as is exaggerate church use in the high density areas, where people have left without a corresponding reduction in the number of churches.

4. The study will cover Denmark where data are amazingly good. The papers are not finished, but the argument made is based on a pilot study.

## References to papers in project on the religious transition

**Theory:** Modeling the link to the theory of economic growth

Gundlach, E., Paldam, M., 2012. A model of the religious transition. *Theoretical Economic Letters* 2012: 416-22

**Part 1: Cross-country dimension:** The religiosity index, *R*, aggregate of 14 items from World Values Survey

Paldam, M., Gundlach, E., 2013. The Religious Transition. A long-run perspective. *Public Choice* 156, 105-23

Paldam, M., Gundlach, E., 2012. Online appendix for: The Religious Transition. A long-run perspective

**Part 2: Long-run time-series dimension:** The church density $\theta$ is proxy for religiosity (p.t. unfinished)

Paldam, E.S., Paldam, M., Paldam, U.S., 2013. Churches in Denmark over 700 years. The effect of the Reformation and the Religious Transition. P.t. incompleate draft

These papers are (will be) available from the home page of the project. The papers refer to the literature.

URL**: http://www.martin.paldam.dk/GT-Religious.php**.

# Appendix: The data used

Table A1.1. The first 40 Municipalities

| (1) Nr | (2) Municipality | (3) Region | (4) *Density* | (5) *Mprc* | (6) *Area* | (7) *Pop* | (8) *Tb* | (9) *All* | (10) *Net* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Aabenraa | *South* | 3.867 | 0.838 | 941.6 | 59,471 | 147,174 | 23 | 23 |
| 2 | Aalborg | *NorJ* | 3.331 | 0.836 | 1144.0 | 201,132 | 150,225 | 67 | 67 |
| 3 | Aarhus | *MidJ* | 2.031 | 0.748 | 468.9 | 315,193 | 158,295 | 64 | 64 |
| 4 | Albertslund | *Cap* | 1.076 | 0.588 | 23.0 | 27,880 | 159,278 | 5 | 3 |
| 5 | Allerød | *Cap* | 2.079 | 0.812 | 67.4 | 24,047 | 219,418 | 5 | 5 |
| 6 | Assens | *South* | 7.247 | 0.899 | 512.0 | 41,394 | 146,233 | 30 | 30 |
| 7 | Ballerup | *Cap* | 1.247 | 0.720 | 34.1 | 48,127 | 181,447 | 6 | 6 |
| 8 | Billund | *South* | 4.562 | 0.882 | 536.5 | 26,303 | 150,537 | 13 | 12 |
| 9 | Bornholm | *Island, Cap* | 5.829 | 0.850 | 588.5 | 41,170 | 140,808 | 24 | 24 |
| 10 | Brøndby | *Cap* | 1.174 | 0.622 | 28.1 | 34,086 | 165,634 | 4 | 4 |
| 11 | Brønderslev | *NorJ* | 5.877 | 0.912 | 633.2 | 35,733 | 137,498 | 21 | 21 |
| 12 | Dragør | *Cap* | 1.457 | 0.816 | 18.1 | 13,723 | 216,349 | 2 | 2 |
| 13 | Egedal | *Cap* | 1.913 | 0.817 | 125.8 | 41,816 | 187,476 | 8 | 8 |
| 14 | Esbjerg | *South* | 3.388 | 0.857 | 742.5 | 115,097 | 152,060 | 39 | 39 |
| 15 | Fanø | *Island, South* | 6.180 | 0.858 | 55.8 | 3,236 | 187,543 | 2 | 2 |
| 16 | Favrskov | *MidJ* | 7.636 | 0.892 | 539.4 | 47,147 | 150,145 | 36 | 36 |
| 17 | Faxe | *Zea* | 6.270 | 0.857 | 404.5 | 35,087 | 153,367 | 22 | 22 |
| 18 | Fredensborg | *Cap* | 1.772 | 0.737 | 112.1 | 39,506 | 199,056 | 7 | 7 |
| 19 | Fredericia | *South* | 2.189 | 0.839 | 134.5 | 50,242 | 157,702 | 11 | 11 |
| 20 | Frederiksberg | *Cap* | 1.389 | 0.648 | 8.1 | 100,814 | 196,991 | 14 | 14 |
| 21 | Frederikshavn | *NorJ* | 4.753 | 0.893 | 648.6 | 61,020 | 149,270 | 30 | 29 |
| 22 | Frederikssund | *Cap* | 4.061 | 0.830 | 305.3 | 44,324 | 172,108 | 18 | 18 |
| 23 | Furesø | *Cap* | 1.310 | 0.721 | 56.7 | 38,174 | 222,107 | 5 | 5 |
| 24 | Faaborg-Midtfyn | *South* | 7.570 | 0.882 | 637.0 | 51,522 | 145,799 | 40 | 39 |
| 25 | Gentofte | *Cap* | 1.372 | 0.721 | 25.5 | 72,890 | 288,553 | 10 | 10 |
| 26 | Gladsaxe | *Cap* | 1.222 | 0.690 | 25.0 | 65,468 | 184,052 | 8 | 8 |
| 27 | Glostrup | *Cap* | 0.923 | 0.745 | 13.3 | 21,672 | 183,923 | 2 | 2 |
| 28 | Greve | *Zea* | 1.668 | 0.786 | 60.2 | 47,975 | 189,287 | 8 | 8 |
| 29 | Gribskov | *Cap* | 3.448 | 0.822 | 280.0 | 40,609 | 181,451 | 14 | 14 |
| 30 | Guldborgsund | *Zea* | 8.907 | 0.853 | 903.4 | 61,750 | 146,671 | 55 | 55 |
| 31 | Haderslev | *South* | 5.336 | 0.876 | 702.0 | 56,219 | 148,242 | 30 | 30 |
| 32 | Halsnæs | *Cap* | 2.591 | 0.811 | 121.2 | 30,873 | 156,593 | 8 | 8 |
| 33 | Hedensted | *MidJ* | 7.393 | 0.893 | 551.5 | 45,989 | 148,548 | 35 | 34 |
| 34 | Helsingør | *Cap* | 1.624 | 0.751 | 121.6 | 61,585 | 179,453 | 12 | 10 |
| 35 | Herlev | *Cap* | 1.503 | 0.704 | 12.0 | 26,607 | 179,341 | 5 | 4 |
| 36 | Herning | *MidJ* | 4.747 | 0.866 | 1323.5 | 86,368 | 147,411 | 41 | 41 |
| 37 | Hillerød | *Cap* | 3.108 | 0.787 | 213.0 | 48,264 | 184,332 | 15 | 15 |
| 38 | Hjørring | *NorJ* | 7.261 | 0.897 | 929.6 | 66,105 | 143,889 | 48 | 48 |
| 39 | Holbæk | *Zea* | 4.758 | 0.822 | 578.7 | 69,354 | 154,980 | 34 | 33 |
| 40 | Holstebro | *MidJ* | 5.768 | 0.883 | 800.2 | 57,217 | 148,775 | 33 | 33 |

Table A1.2. The next 40 Municipalities

| (1) Nr | (2) Municipality | (3) Region | (4) *Density* | (5) *Mprc* | (6) *Area* | (7) *Pop* | (8) *Tb* | (9) *All* | (10) *Net* |
|---|---|---|---|---|---|---|---|---|---|
| 41 | Horsens | *MidJ* | 4.051 | 0.827 | 542.0 | 83,921 | 144,419 | 34 | 34 |
| 42 | Hvidovre | *Cap* | 0.786 | 0.708 | 24.9 | 50,871 | 169,738 | 5 | 4 |
| 43 | Høje-Taastrup | *Cap* | 1.455 | 0.678 | 78.4 | 48,114 | 167,050 | 7 | 7 |
| 44 | Hørsholm | *Cap* | 1.233 | 0.800 | 31.4 | 24,330 | 275,197 | 3 | 3 |
| 45 | Ikast-Brande | *MidJ* | 4.431 | 0.869 | 736.4 | 40,625 | 141,556 | 19 | 18 |
| 46 | Ishøj | *Cap* | 1.421 | 0.551 | 25.9 | 21,111 | 150,294 | 3 | 3 |
| 47 | Jammerbugt | *NorJ* | 8.020 | 0.897 | 872.9 | 38,651 | 146,558 | 31 | 31 |
| 48 | Kalundborg | *Zea* | 7.826 | 0.867 | 604.0 | 48,555 | 154,725 | 38 | 38 |
| 49 | Kerteminde | *South* | 6.317 | 0.887 | 205.9 | 23,746 | 151,673 | 15 | 15 |
| 50 | Kolding | *South* | 3.575 | 0.842 | 605.0 | 89,523 | 156,067 | 32 | 32 |
| 51 | København | *Cap* | 1.487 | 0.585 | 74.4 | 551,580 | 167,495 | 86 | 82 |
| 52 | Køge | *Zea* | 2.784 | 0.810 | 255.5 | 57,467 | 163,075 | 16 | 16 |
| 53 | Langeland | *South* | 15.366 | 0.886 | 291.2 | 13,016 | 151,998 | 20 | 20 |
| 54 | Lejre | *Zea* | 6.308 | 0.830 | 240.1 | 26,950 | 177,681 | 17 | 17 |
| 55 | Lemvig | *MidJ* | 12.192 | 0.920 | 508.2 | 21,326 | 150,156 | 26 | 26 |
| 56 | Lolland | *Zea* | 11.950 | 0.846 | 891.9 | 45,190 | 147,716 | 54 | 54 |
| 57 | Lyngby-Taarbæk | *Cap* | 1.312 | 0.739 | 38.9 | 53,357 | 229,623 | 7 | 7 |
| 58 | Læsø | *Island, NorJ* | 15.957 | 0.916 | 118.0 | 1,880 | 145,813 | 3 | 3 |
| 59 | Mariagerfjord | *NorJ* | 7.789 | 0.890 | 792.9 | 42,370 | 144,720 | 33 | 33 |
| 60 | Middelfart | *South* | 5.584 | 0.892 | 299.9 | 37,607 | 155,848 | 21 | 21 |
| 61 | Morsø | *NorJ* | 15.913 | 0.863 | 367.7 | 21,366 | 141,998 | 34 | 34 |
| 62 | Norddjurs | *MidJ* | 10.034 | 0.887 | 721.2 | 37,872 | 144,037 | 38 | 38 |
| 63 | Nordfyns | *South* | 10.226 | 0.904 | 451.6 | 29,337 | 145,250 | 30 | 30 |
| 64 | Nyborg | *South* | 6.045 | 0.860 | 276.2 | 31,429 | 151,316 | 20 | 19 |
| 65 | Næstved | *Zea* | 5.299 | 0.848 | 681.0 | 81,149 | 153,719 | 43 | 43 |
| 66 | Odder | *MidJ* | 6.894 | 0.869 | 225.1 | 21,758 | 159,185 | 15 | 15 |
| 67 | Odense | *South* | 1.928 | 0.780 | 304.3 | 191,903 | 148,591 | 37 | 37 |
| 68 | Odsherred | *Zea* | 4.608 | 0.864 | 355.3 | 32,554 | 158,307 | 16 | 15 |
| 69 | Randers | *MidJ* | 6.160 | 0.872 | 800.1 | 95,776 | 144,835 | 59 | 59 |
| 70 | Rebild | *NorJ* | 11.081 | 0.893 | 625.0 | 28,879 | 147,484 | 32 | 32 |
| 71 | Ringkøbing-Skjern | *MidJ* | 8.462 | 0.900 | 1488.9 | 57,904 | 155,007 | 49 | 49 |
| 72 | Ringsted | *Zea* | 5.434 | 0.813 | 295.5 | 33,127 | 155,189 | 18 | 18 |
| 73 | Roskilde | *Zea* | 2.285 | 0.784 | 211.9 | 83,148 | 185,364 | 20 | 19 |
| 74 | Rudersdal | *Cap* | 1.465 | 0.770 | 73.8 | 54,626 | 266,115 | 8 | 8 |
| 75 | Rødovre | *Cap* | 1.083 | 0.711 | 12.1 | 36,920 | 171,862 | 4 | 4 |
| 76 | Samsø | *Island, MidJ* | 18.102 | 0.833 | 114.3 | 3,867 | 156,221 | 7 | 7 |
| 77 | Silkeborg | *MidJ* | 4.254 | 0.864 | 864.9 | 89,329 | 155,771 | 38 | 38 |
| 78 | Skanderborg | *MidJ* | 4.999 | 0.870 | 436.1 | 58,013 | 158,979 | 29 | 29 |
| 79 | Skive | *MidJ* | 9.870 | 0.905 | 690.7 | 47,620 | 147,636 | 47 | 47 |
| 80 | Slagelse | *Zea* | 5.701 | 0.833 | 567.3 | 77,185 | 150,693 | 44 | 44 |

Table A1.3. The last 18 Municipalities

| (1) Nr | (2) Municipality | (3) Region | (4) *Density* | (5) *Mprc* | (6) *Area* | (7) *Pop* | (8) *Tb* | (9) *All* | (10) *Net* |
|---|---|---|---|---|---|---|---|---|---|
| 81 | Solrød | *Zea* | 2.839 | 0.838 | 40.0 | 21,135 | 195,700 | 6 | 6 |
| 82 | Sorø | *Zea* | 6.482 | 0.860 | 311.1 | 29,314 | 155,203 | 19 | 19 |
| 83 | Stevns | *Zea* | 8.706 | 0.855 | 250.2 | 21,823 | 162,102 | 19 | 19 |
| 84 | Struer | *MidJ* | 7.707 | 0.883 | 250.8 | 22,057 | 148,796 | 17 | 17 |
| 85 | Svendborg | *South* | 5.466 | 0.831 | 417.0 | 58,545 | 146,774 | 32 | 32 |
| 86 | Syddjurs | *MidJ* | 8.600 | 0.870 | 696.3 | 41,862 | 152,001 | 37 | 36 |
| 87 | Sønderborg | *South* | 3.682 | 0.847 | 496.6 | 76,037 | 151,903 | 28 | 28 |
| 88 | Thisted | *NorJ* | 13.151 | 0.883 | 1101.7 | 44,864 | 142,415 | 59 | 59 |
| 89 | Tønder | *South* | 8.196 | 0.877 | 1278.0 | 39,043 | 145,372 | 32 | 32 |
| 90 | Tårnby | *Cap* | 0.972 | 0.775 | 65.0 | 41,168 | 176,399 | 4 | 4 |
| 91 | Vallensbæk | *Cap* | 1.370 | 0.714 | 9.2 | 14,603 | 180,587 | 2 | 2 |
| 92 | Varde | *South* | 6.579 | 0.902 | 1255.8 | 50,156 | 153,603 | 33 | 33 |
| 93 | Vejen | *South* | 6.311 | 0.875 | 814.4 | 42,784 | 137,575 | 27 | 27 |
| 94 | Vejle | *South* | 4.344 | 0.839 | 1066.3 | 108,186 | 156,418 | 47 | 47 |
| 95 | Vesthimmerland | *NorJ* | 10.640 | 0.899 | 771.8 | 37,595 | 137,550 | 40 | 40 |
| 96 | Viborg | *MidJ* | 7.874 | 0.881 | 1474.1 | 93,984 | 151,275 | 74 | 74 |
| 97 | Vordingborg | *Zea* | 5.905 | 0.848 | 621.0 | 45,725 | 148,136 | 27 | 27 |
| 98 | Ærø | *Island, South* | 10.553 | 0.877 | 90.5 | 6,633 | 140,411 | 7 | 7 |

Figure 3 shows that most of the 30 members of the left hand group are from the Capital Region, and most of the 68 members of the right hand group are in other regions. Some exceptions occur in each group. They are listed in Table A4. Some large towns (Fredericia, Odense and Aarhus) are in the left hand group. The remaining four exceptions are borderline cases. It is arguable that Greve is a Copenhagen suburb and that Frederikssund, Gribskov and Hillerød rather belong to the Zealand Region.

Table A2: Exceptions to groups for *Lndens*

| Non-Cap in left hand group | | Cap in right hand group | |
|---|---|---|---|
| Municipality | Region | Municipality | Region |
| Fredericia | South | Frederikssund | Cap |
| Greve | Zea | Gribskov | Cap |
| Odense | South | Hillerød | Cap |
| Aarhus | MidJ | | |